Can the BRICS+ become a sustainable economic and financial union?*

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Abstract

This paper addresses the following question. Could the BRICS countries have formed a sustainable economic and monetary union (EMU) if they had decided to do so in the early 2000s? Our sustainability criterion is that the macroeconomic imbalances of the countries should not deviate too far from each other. We consider a convergence-based approach to quantify such a situation. We investigate a scenario where each country's bilateral nominal exchange rate is defined in relation to a central rate defined by considering a basket of the various BRICS national currencies. We find that the group would not have formed a sustainable EMU from the late 2000s onwards. We conclude that this makes the project of de-dollarizing their respective economies not necessarily viable, unless the group choose a proper exchange rate regime

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1.- Introduction

Could the countries that make up the BRICS group, after the enlargement in January 2024, have formed a viable economic and monetary union over the long term? We interpret the notion of viability to as a concept related to macroeconomic and financial sustainability. We look at the capacity countries would have had to ensure a convergence of their macroeconomics and financial fundamentals, which we summarize in an indicator, i.e. the equilibrium real exchange rate. More specifically, this paper proposes an analysis of the convergence of countries' misalignments based on a fictitious scenario in which we assume that the BRICS group would have been enlarged in the early 2000s, and that the member countries would then have decided to define the fluctuation margins of their currencies in relation to a synthetic currency made up of each of their currencies. We find this question interesting for the following reason.

The original group of the BRICS has been made up of Brazil, Russia, India, China and South Africa. Since January 2024, it has expanded to include 5 new members (Saudi Arabia, United Arab Emirates, Egypt, Iran and Ethiopia). We call the new group of 10 countries the BRICS+. From a geopolitical viewpoint, this enlargement is part of the so-called "Global South" project, which aims to counterbalance the political and economic weight of Western industrialized countries, specifically the United States and Europe. The project of rebalancing of international relations includes the unofficial objective of de-dollarizing the world, in the face of growing opposition to the predominance of the US dollar in international transactions. In this regard, the BRICS+ countries have signed a number of agreements with each other to use their own currencies in bilateral trade transactions. This strategy could be accentuated by the growing role of the New Development Bank (BRICS Development Bank) in financing development infrastructure projects.

Is it possible to conceive of the BRICS+ as a new entity in global economic governance capable of competing with the predominance of the US dollar in the current international monetary system? The idea developed in this paper is that such a project might be feasible if the BRICS+ form a sustainable economic and financial union. Our proposed sustainability criterion is the following : global imbalances between member countries should not be too large. We propose a framework and look for conditions under which actual historical data would had led to a sustainable union, had the countries decided to do so. The framework is built in such a way as comparing a synthetic indicator of internal and external imbalances (real exchange rate misalignments relative to its equilibrium value) if the countries had decided to build an

economic and financial union, with the rule that the indicators of macroeconomic imbalances should not diverge from each other by more than a certain threshold. To construct our scenario, we propose a multilateral framework in which the equilibrium real exchange rate of each country is evaluated against a common currency constructed from a basket of currencies of the member countries according to their economic and financial weight. Furthermore, we assume the existence of a rule for limiting macroeconomic imbalances between member countries: the gap between each country' misalignment and the average misalignment of the BRICS should not exceed a certain threshold. If it does, the misalignment is deemed excessive. Our goal is then to calculate, for each country, the proportion of excessive misalignment episodes, had the countries adopted our counterfactual framework since the early 2000s (when the BRICS project first took shape between Brazil, Russia, India and China).

There are several reasons for targeting a level of misalignments relative to those of other countries. Firstly, bringing misalignments closer to those of other countries avoids excessive inflation rate differentials and differences in volatility. Indeed, we know that divergences in inflation rates can occur as a result of pass-through from the real exchange rate to the inflation rate. Secondly, several of the countries that have recently joined the BRICS have strong financial powers (e.g. Saudi Arabia and the United Arab Emirates). The vocation of the BRICS group is to create a financial market with a significant volume of liquidity to finance the economic development of member countries. The financial literature suggests that investors' comparative knowledge of countries' macro-financial fundamentals is an important element in portfolio arbitrage (for an example, see Dahlquist and Hasseltoft, 2020). Finally, knowledge of relative misalignments between countries can be considered a synthetic leading indicator of differences in output-gaps, inflation rates, unemployment rates, etc.

Our contribution to the literature is to investigate the question of macroeconomic sustainability of a group of countries from the angle of the convergence of their real exchange rate misalignments, as a proxy mechanism for measuring the degree of heterogeneity of their macroeconomic and financial policies. Indeed, the question of sustainability has often been approached from other perspectives. For instance, there exists a voluminous literature on risk-sharing and resilience to asymmetric and the asynchronous nature of business cycles. This literature suggests that business cycle mismatches caused by idiosyncratic asymmetric shocks can be corrected either by capital transfers (provided capital markets are sufficiently developed) or by private and public transfers along the lines of the insurance mechanisms set up in federal states. Other works consider the traditional criteria of optimum currency areas to assess limited

divergence between the countries market developments. When sustainability is interpreted in terms of the ease with which markets adjust to counter the adverse effects of asymmetric shocks, the authors focus on the degree of mobility of factors of production (labour, physical and financial capital).

The empirical framework we propose is based on the following narrative. To build a credible alternative to the dollar, which is one of the BRICS+' medium-term objectives, several conditions will need to be satisfied. Firstly, even if countries do not immediately wish to use a common currency, they will at least have to choose to carry out their transactions and increase the proportion of their foreign exchange reserves in BRICS' currencies. This could be done by defining a basket of their national currencies for their trade and financial transactions. Secondly, these currencies should serve as safe assets, so that international investors would want them in their portfolio choices. These are necessary, but not sufficient, conditions. For example, the history of the euro zone shows that the adoption of a common currency has not prevented the member countries from diverging from one another. The question of macroeconomic and financial imbalances is therefore fundamental for the macroeconomic and financial situation of a group of countries to be considered credible, and if speculative destabilization is to be avoided.

Our main results are as follows. We apply a model of beta-convergence and sigma-convergence to highlight a divergence of real exchange rate misalignments between the members of the BRICS+ group, if the countries had taken the decision from the early 2000s to make their economic policies convergent by monitoring the evolution of their respective real exchange rates and the evolution of their deviation from an average equilibrium exchange rate common to their group. Three countries stand out as "outsiders" in the misalignment convergence dynamic: Iran, Ethiopia and Egypt. Their real exchange rates would have been significantly overvalued compared with those of the other members. And the divergence from other countries would have increased over the years. Moreover, China, which was part of the original BRICS group, would have seen its real exchange rate and exchange rate misalignment converge closer to that of the UAE and Saudi Arabia than to that of the other historic BRICS members (Russia, India, Brazil, South Africa). Convergence tests show that individual convergence of real exchange rate misalignments would have been too slow to reduce the existing gaps between countries in the medium term.

The remainder of the paper is structured as follows. Section 2 presents the empirical framework proposed to investigate the sustainability of the BRICS+ group. In Section 3, we provide evidence of a divergence of misalignments of real exchange rates. Finally, Section 4 concludes.

2.- A framework to analyse the sustainability of a BRICS+ economic union

We use the equilibrium real exchange rate as an indicator of a country's internal and external balances. Contrary to a common approach in the literature, the aim here is not for each country to minimize the gap between its exchange rate and its own equilibrium exchange rate. Instead, we propose a criterion for minimizing the gap between countries' misalignments. This is equivalent to considering that the misalignment target for a given country is not zero, but an average (to be defined) of the other member countries' misalignments, from which individual countries should not deviate too much. There are several reasons for doing this.

Firstly, by not forcing countries to minimize their own misalignment, we can alleviate the adjustment constraints required to bring the BRICS+ members structurally closer to each other. The example of the Eurozone shows that imposing the same numerical targets on macroeconomic variables (for example, when adjusting fiscal policies or public debt) can lead to unstable equilibria, because countries' structural divergences are not considered. The divergences can be so great as to prevent the realization of the law of one price on goods and services markets, or financial markets. It therefore seems difficult, for example, to require countries to have the same levels of output gap or current account balances.

Our second motivation is that the case of the BRICS+ countries is different from that of the eurozone countries. Indeed, the adoption of a single currency forces the countries to make their economic fundamentals converge towards each other in a strict manner. In the scenario considered in this paper, each BRICS+ member country keep its own currency. The deviations in their fundamentals should simply reflect the deviations in the respective exchange rates of the currencies against each other (or the exchange rate of each currency against an anchor currency - a numeraire currency - defined by a basket of currencies made up of the different currencies). The weight of each currency in this basket depends on the respective macroeconomic and financial weights.

Our methodology, to compare the differences between countries' macroeconomic imbalances, involves two steps. Firstly, we define the equilibrium real exchange rates and,

secondly, we propose a criterion of global misalignment of the exchange rate within the BRICS+.

2.1. - Econometric framework : an ARDL BEER model of real exchange rate

The study of equilibrium exchange rates is usually motivated in the literature from the point of view of portfolio choice theories, giving investors in foreign exchange markets advanced indicators of exchange rates and guiding them in their trade-offs (see, for example, Colacito et al. 2020, Rubaszek et al. 2023). In the macroeconomic literature, the motivation for estimating equilibrium exchange rates is also to prevent persistent imbalances in a country's main macro-financial indicators (for a recent paper, see Kharrat et al. 2020). Member countries of an economic union have an additional constraint, in that they must also ensure that misalignments within the group are not too different from one another.

Several approaches for measuring the equilibrium exchange rates and their misalignments have been proposed in the literature (for a survey, the reader can refer to Bussière et al., 2020). Among the different proposals, we choose the Behavioral Equilibrium Exchange Rate (BEER), a regression approach introduced in seminal papers by Clark et al. (1994), Clark and MacDonald (1999). Our approach focuses on global imbalances and bilateral balance of payments imbalances (current and financial accounts). The issue at stake is whether or not bilateral currency parities reflect persistent balance-of-payments imbalances between BRICS+ countries. To our knowledge, there are few works in the literature that have examined the links between exchange rate misalignments and balance of payments imbalances. Researchers usually focus on the link between current accounts and the real exchange rate, but only a few consider causal relationships between misalignments and current accounts (among them, Camba-Crespo et al. 2022, Arghyrou and Chortareas 2008, for Eurozone countries, Gnimassoum and Mignon (2015) on a sample of 22 industrialized countries, Comunale 2018 for central European countries). Moreover, in most studies, causality runs from misalignments to current accounts.

The importance of not considering current accounts alone is that capital flows can explain exchange rate misalignments (financial channel via portfolio behavior), and this leads governments to trade-off between an internal equilibrium objective and an external equilibrium objective (See Corsetti et al. 2018).

The question of how many independent variables should be considered in a BEER model has led to two lines of research. The first is to consider as many variables as possible, so as to reproduce real exchange rate trajectories as closely as possible. The criterion here is to be as exhaustive as possible in the selection of the variables. An alternative approach is to keep to a parsimonious model, with a few regressors. Ca'Zorzi and Rubaszek (2023) have shown that models with a very large number of variables do not give better ex-post forecasts of exchange rate trajectories than very simple models with a few important explanatory variables (three) or a simple PPP model. There is therefore a risk of over-parameterization when selecting models using a data-mining approach. In our case, the explanatory variables we consider therefore relate to some key determinants of current account balances and the financial account. We introduce variables considered to represent a basic model (with terms of trade, relative productivities, foreign exchange reserves, etc.), in addition to variables specific to the BRICS+ group (differentials between their interest rates, between their inflation rates, the share of trade between them, etc.).

The econometric framework we choose is a standard ARDL model. What changes is the definition of the independent and the explanatory variables. The econometric equation is an $ARDL(p, q_1, \dots, q_j)$ model in error-correction form:

$$\Phi(B,p)\Delta e_{i,t} = \sum_{j=1}^{K} \Theta_j(B,q_j)\Delta X_{i,t}^{j} + \left(\lambda_0 e_{i,t-1} + \sum_{j=1}^{K} \lambda_j X_{i,t-1}^{j}\right) + \varepsilon_{i,t}, \quad , \quad (1)$$

$$\text{where} \quad \Phi(B,p) = \sum_{\tau=0}^{p} \phi_\tau B^\tau, \;\; \Theta_j(B,q_j) = \sum_{\kappa=0}^{q_j} \theta_{j,\kappa} B^\kappa, \quad \varepsilon_{i,t} \sim iid(0,\sigma_\varepsilon^2). \qquad (2)$$

$e_{i,t}$ is the real exchange rate of country i at time t for $i = 1, \dots, N, \quad t = 1, \dots, T$ (see the definition below). We assume that there are K macro-financial fundamentals $X^j, j = 1, \dots, K$. B is the backward shift operator defined by $B^\tau Z_t = Z_{t-\tau}$. The autoregressive terms of the independent variables are defined up to p lags and the X^j explanatory variables respectively have up to q_j lags. The equation assumes that the explanatory variables are weakly exogenous with respect to the independent variable, and that there is only one long-term relationship. We use the latter to define the equilibrium (long-term) real exchange rate.

2.2.- Dependent variable : CPI-based real bilateral exchange rates of individual BRICS+ countries

We define the independent variable in Equation (1) as the product of a country's nominal exchange rate vis-à-vis a virtual BRICS+ common currency and the ratio of prices between the BRICS+ and domestic country:

$$e^{brics} = s^{brics}\frac{p^*}{p},$$

where e^{brics} is the bilateral exchange real exchange rate, s^{brics} is the nominal bilateral exchange rate, p* is the average CPI (consumer price index) within the BRICS+ except the country in consideration, and p is domestic CPI.

Computation of the bilateral nominal exchange rate s^{brics}.

a) We start by defining a basket of BRICS+ currencies. Let us take a virtual currency called brics, whose value against the dollar we want to define, and which corresponds to the weighted average of the exchange rates of the BRICS+ currencies:

$$s^{brics} = \prod_{k=1}^{K}(s^k)^{\theta_k},\ \theta_1 + \theta_2 + \ldots + \theta_K = 1,\ 0 \le \theta_k \le 1. \quad (3)$$

s^k is the price of domestic currency in country k per unit of dollar, θ_k is the weight of country k is the basket.

To compute the weights, we consider the sum of each country's share of bilateral trade with the other BRICS+ members, GDP share in total of the BRICS+'s GDP, and each country foreign reserves in the total of the BRICS+'s foreign reserves. (see Appendix 2 for a detailed description of the computation of the weights).

b) We apply the formula in Equation (3), which gives us an average nominal exchange rate of the BRICS+ currency vis-à-vis the US dollar .

c) Once we know what is a country k's nominal exchange rate vis-à-vis the dollar, and what is average exchange rate for the BRICS vis-à-vis the US dollar, we can deduce each country bilateral nominal exchange rate against the BRICS+ currency (s^{brics})

d) Then we compute e^{brics}.

2.3.- Independent variables

The explanatory variables are chosen so as to have the largest amount of data available for the maximum number of BRICS+ members. The detailed source of data in Appendix 2.

- *Relative productivity*, defined as the ratio of each country productivity and the average of the productivities of the other BRICS member. A common interpretation is that productivity differentials in a real exchange rate equation capture a Balassa-Samuelson effect. This is valid if several assumptions are satisfied (the share of the tradable and non-tradable goods sectors is identical in the domestic and foreign countries, international competition causes the prices of tradable goods to converge, productivity differences between countries concern the non-tradable goods sector). In this case, any positive productivity differential between a country and a foreign country leads to an appreciation of its currency. In our case, these hypotheses are unlikely to be verified, notably due to the heterogeneity of the sectoral structures of the BRICS+ member countries (the degrees of integration into international trade are different, and the law of one price for those whose economies are subject to international competition is unlikely to apply.

We consider an alternative interpretation. Productivity differentials capture a Penn effect. The latter has been highlighted in empirical literature to underline the existence of a positive link between a country's per capita income and the general price level (for a survey of empirical studies, see Fujii, 2015). A rise in overall productivity, whatever the sector, increases GDP per capita (TFP being a factor that drives long-term growth in growth models). The effect on the general price level is explained by the fact that the resulting rise in per capita income increases demand and raises the price of non-tradable goods. In our equation, productivity differentials therefore capture the influence of differences in living standards between BRICS+ countries on the bilateral real exchange rate.

- *Bilateral trade flows*, defined by the share of a country's trade with the other BRICS+ members, relative to total trade between all members. Trade is measured by the sum of bilateral exports and imports. The links between real exchange rates and trade flows are often examined in a causal sense, from the former to the latter. This is particularly true of work on price competitiveness. In the opposite causal direction, which is the one adopted, the real exchange rate is a variable that adjusts to re-establish the current account balance over the medium term. A current account deficit requires monetary depreciation to return to a balanced balance, while current account surpluses cause the exchange rate to appreciate. This movement takes place through variations in the nominal exchange rate. To limit endogeneity problems, we have chosen bilateral

trade flows as a "proxy" for current account balances. They capture the relative positions of current balances between the BRICS+ countries.

- *Terms of trade* defined as the ratio of exports and import prices. The terms of trade effect captures several channels. The first is a supply channel via the costs of imported goods and the profits of exporting sectors. Import and export price movements influence the margin behaviour of exporting and importing companies when they set their prices on domestic and international markets. The second is a demand channel. Indeed, the terms of trade can explain the asynchronous nature of business cycles between the BRICS+ countries. This affects relative prices between countries.
- *Inflation differentials* between a country's inflation and the weighted average inflation of the other BRICS+ members. We want to understand the extent to which inflation rate differentials reflect real exchange rate differences between the BRICS+. This variable also allows to account for rigidities in the adjustment of prices in goods and labour markets.
- *Interest rate differentials*. Differences between domestic real interest rate on long-term interest rate and the weighted average of BRICS members' real long-term interest rates. This variable captures frictions in capital mobility across the BRICS+ countries, i.e. uncovered parity interest rates.
- The ratio of a country's *reserve assets to nominal GDP*.
- Debt ratio as share of GDP.

The regression of the variable e^{brics} on the exogenous variables listed above is based on an ARDL model, using quarterly data from 2000q1 to 2022q4. For each country, the equilibrium exchange rate is computed by the fitted values of the real exchange rate in the cointegration relationship derived from the ARDL model.

2.4.- Main determinants of real exchange rates for the BRICS+

To avoid overloading the paper with tables, ARDL regression results are not included in the paper, but are available on request from the authors. We report below in Table 1 the sum of the significant coefficients of the estimated ARDL models. The coefficients show heterogeneity in the effects of the different variables on the real exchange rate, with signs and coefficient sizes varying between countries. Our main findings are the following.

Most often, relative productivity leads to a depreciation of the exchange rate (positive coefficient), except in Egypt and Iran. This leads us to take with caution the systematic validation of the so-called "productivity bias" hypothesis (Balassa-Samuelson effect) for emerging and developing economies. In theory, the effect of a productivity on the equilibrium exchange rate depends on the distribution of productivity gains between the tradable and non-tradable goods sectors. If they are equally distributed between these two sectors, there is no effect. It is then possible to find non-significant coefficients, as is the case here for Ethiopia and the United Arab Emirates. The positive relationship may turn to negative (leading a depreciation as found here for the majority of countries) in several circumstances: the concentration of productivity gains in the service sector, the hypothesis of home-bias in the distribution sector when products consumed at home and abroad are different, the existence of price differentiation behavior on the part of firms). For examples, see Benigno and Thoenissen (2003), MacDonald and Ricci (2001), Lee and Tang (2007), Peltonen and Sager (2009).

The terms of trade coefficient is mostly negative. A shock that raises export prices and/or lowers import prices leads to a real appreciation. This illustrates the existence of a correlation between the price of commodities exported by developing countries and the exchange rate. Notably, in the sample, we have major oil and gas exporters (Iran, Saudi Arabia, Russia), the others exporting agricultural commodities and industrial raw materials. The transmission channel may be foreign exchange reserves, which increase with commodity terms of trade (ratio between the price of exported commodities and imported manufactured goods). This result is in line with similar findings by other authors. The interested readers can refer to, among others, Bodart et al. (2015), Cashin et al. (2004), Coudert et al. (2008), Kohlscheen et al. (2017).

Higher domestic inflation than in other countries increases demand for foreign goods, which should lead to a depreciation of the domestic currency (in this case, a positive coefficient on the inflation rate differential variable). We only obtain a positive sign for three countries (Brazil, Iran and Russia). The negative sign obtained for 5 other countries (Egypt, Ethiopia, India, Saudi Arabia, South Africa) may illustrate the case of a real exchange rate appreciation that coexists with a dual inflation system, i.e. a pronounced divergence between the price of tradable and non-tradable goods (the latter may adjust slowly to the former in emerging economies, if there are nominal rigidities and frictions in capital accumulation, as shown, for example, by Vilagi, 2005).

For more than half the countries in the sample, there is weak evidence of a correlation between the real interest rate differential and the real exchange rate (insignificant coefficients). Where it is significant, it is usually positive. The interest rate differential can be considered as a proxy for the expected real depreciation of the BRICS common currency, which is equivalent to testing an uncovered interest rate parity condition. If it is significant and consistent with the flexible price hypothesis, then the interest rate differential should be negatively correlated with the real exchange rate, i.e. lead to an appreciation of the latter, as we find here in the case of India. Indeed, a higher interest rate relative to the average of the BRICS+ leads an inflow of capital that induce an appreciation of the nominal interest rate. However, when prices are sticky, raising the domestic rates does not lower prices to the full extent expected, which can in turn expect the finding of a positive correlation with the real exchange rate (as here in the case of Iran and Russia).

International reserves are usually used as a hedging instrument to protect emerging and developing countries against negative external shocks (negative terms-of-trade shocks or sudden stops). A high level of reserves leads to an appreciation of the real exchange rate, and so in this scenario where reserves are held as buffers, the expected sign of the coefficient is negative (as in the case of Egypt, Ethiopia and India). On the other hand, the self-insurance motive becomes secondary, as governments may focus on maintaining the competitiveness of their economy and intervene on the foreign exchange market to prevent a real appreciation of their currency. A positive correlation between reserves and the real exchange rate may mean that governments prefer to let their currencies depreciate rather than draw on their foreign exchange reserves. This is the case in Iran, China and South Africa, where a higher level of international reserves has a significant positive effect on the real exchange rate. We find the ambiguous effects of international reserves on the real exchange rate highlighted in the empirical literature devoted to emerging and developing countries (see, for example, Aizenman et al. 2012, Aizenman et al. 2024, BIS Papers 2019, Choi and Taylor 2022, Gosh et al. 2014, Pina 2015).

Table 1. Sum of significant coefficients (at 10% level of confidence) in ARDL models

	Brazil	**China**	**Egypt**	**Ethiopia**	**India**
Relative productivity	0.101	0.01	-1.176	-	1.686
Trade share	-	-	-	7.20	-1.899
Terms of trade	-	-	-	-0.882	-1.781
Inflation diff.	0.02	-	-0.01	-0.02	-0.02
Interest rate diff.	-	-	-	0.007	-0.02
Reserves	-	0.19	-0.247	-0.16	-0.46
Debt ratio	-0.126	0.113	0.215	-	-1.272
	Iran	**Russia**	**Saudi Arabia**	**South Africa**	**UAE**
Relative productivity	-0.07	0.125	0.83	0.811	-
Trade share	-0.28	2.412	0.551	2.52	2.657
Terms of trade	-0.34	0.04	-0.12	-0.182	-
Inflation diff.	0.01	0.01	-0.008	-0.017	-
Interest rate diff.	0.01	0.01	-	-	-
Reserves	0.03	-	-	0.201	-
Debt ratio	-	0.09	0.01	0.275	0.012

3.- The sustainability of a BRICS+ zone : an approach based on convergence

We define a country i's exchange rate misalignment as the error-correction term, i.e. the residuals of the long-run relationship measured contemporaneously:

$$y_{it} = e_{it}^{brics} - EER_{it},\ i = 1, \ldots, 10, \qquad (4)$$

where EER_{it} is country i's equilibrium exchange rate.

We propose the following definition of sustainability. The BRICS+ group is economically sustainable if the internal and external imbalances captured by exchange rate misalignments are not too different between countries. In other words, countries may have heterogeneous internal and external macroeconomic imbalances. We do not focus – as is usually the case in the literature – on the deviations from their own equilibrium exchange rate. What matters is that misalignments do not diverge too widely *between* the BRICS members. To measure such a deviation, we propose two approaches.

3.1.-Using beta-convergence and sigma convergence indicators

We propose an approach based on convergence indicators: b-convergence, sigma convergence and delta-convergence. Assuming that each country, before comparing itself with the others, tries to have the smallest possible misalignments (i.e. to be the best possible at reducing its own imbalances), the concept of beta-convergence allows us to see whether or not each country is getting closer to the one that is doing best in minimizing its misalignments. In the following convergence equation, we can therefore take as our reference country the one that does best in minimizing its misalignments relative to the others.

A convergence coefficient is estimated using a standard β-convergence equation:

$$y_{it} - y_{bt} = \alpha + \beta(y_{it-1} - y_{bt-1}) + \varepsilon_{it}, \quad i = 1, \dots, 10, \quad t = 2004q1, \dots, 2022q4. \quad (5)$$

α is a constant and $\varepsilon_{it} \sim iid(0, \sigma_\varepsilon^2)$ is a residual term. y_{it} is country i's misalignment at time t, y_{bt} is the misalignment of the best performer (in terms of minimizing its own misalignment) at time t. If we are interested by beta-convergence in the sense we have just defined it, then y_{bt} must be the absolute value of the country misalignment closest to zero in the sample at time t. Similarly, we need to take the absolute value of y_{it} to know whether or not a country is far from the best performer.

Figure 1 shows the misalignments calculated from ARDL model estimates. Deviations are expressed as percentages. Looking at this graph, we can see that in some years, countries have very significant misalignments of up to -15% in the case of undervaluation of the real exchange rate, and up to 20% in the case of overvaluation over the whole period. Table 2 shows, over all quarters, the frequency with which a given country would have been selected as the best performer. The two top performers are Brazil and Iran, while the countries that would be least frequently selected are Saudi Arabia and Russia. Finally, countries such as Ethiopia and Egypt would be chosen as often as China as best performers. The numbers suggest that the countries selected for each quarter changes a lot between quarters.

This table therefore shows the limitations of choosing a country that minimizes its misalignment as a benchmark. Indeed, it is unreasonable to consider that the BRICS+ align their economic policies with those of the least developed countries in the group (both in terms of financial markets and macroeconomic development indicators). Besides, we cannot compare the position of China, which would appear to be a leading country in the conduct of BRIC+ economic policies, with that of Egypt or Ethiopia.

It is therefore better, if we adopt the beta-convergence approach, to choose, for example, a weighted average of the misalignments. The weights are defined weighted by the GDP size relative to the BRICS+'s GDP as a whole. Taking this last criterion, we see that China and Brazil are the two countries that come closest to the best performers, which is more satisfying.

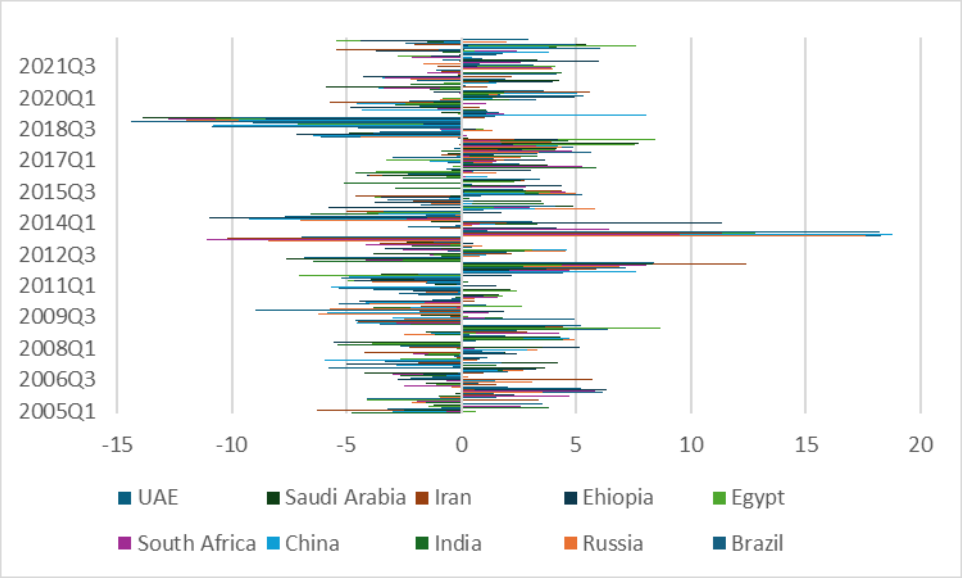


Figure 1. Misalignment in percent

Table 2. How often would a country be chosen as the best performer?

Criterion: Smallest misalignments									
Brazil	Russia	India	China	South Africa	Egypt	Ethiopia	Iran	Saudi Arabia	UAE
15.28	5.56	11.11	8.33	11.11	8.33	8.33	16.67	4.17	11.11
Criterion : closest to the weighted average of all countries' misalignments									
Brazil	Russia	India	China	South Africa	Egypt	Ethiopia	Iran	Saudi Arabia	UAE
13.89	8.33	9.72	22.22	9.72	5.56	8.33	8.33	5.56	8.33

To see whether misalignment gaps are narrowing (widening), we need that $|\beta| < 1$ ($|\beta| \geq 1$). Let us define $sp_{it} = y_{it} - y_{bt}$. Denoting σ^2_{spt} the empirical variance of the misalignment gap, Equation (5) implies

$$\sigma_{spt}^2 = \beta^2 \sigma_{spt-1}^2 + \sigma_\varepsilon^2, t = 1, \dots, T, \quad (6)$$

A general solution of this equation takes the form

$$\sigma_{spt}^2 = \sigma_{spt}^{2*} + \beta^{2t}\left[\sigma_{sp0}^2 - \sigma_{sp0}^{2*}\right], \quad \sigma_{spt}^{2*} = \frac{\sigma_\varepsilon^2}{1-\beta^2}, t = 1, \dots, T. \quad (7)$$

σ_{spt}^{2*} is the steady state value of σ_{spt}^2. If $0 < |\beta| < 1$, then $\sigma_{spt}^2 \to \sigma_{spt}^{2*}$ when $t \to +\infty$. If σ_{spt}^2 is ergodic, then σ_{spt}^{2*} is bounded. If not, $\sigma_{spt}^{2*} \to +\infty$. Therefore beta-convergence may happen without sigma-convergence. Indeed, it is possible that the speed of convergence is not enough to eventually lead to a reduction in the dispersion of misalignments.

This leads us to propose a first definition of the sustainability of a BRICS+ EMU.

Definition 1 of sustainability. The BRICS+ form a sustainable EMU beta-convergence occurs $(0 < |\beta| < 1)$ and if σ_{spt}^2 is ergodic, i.e the moment of the distribution of σ_{spt}^2 must converge as new observations are added to the series). Otherwise, it is unsustainable.

Beta-convergence may well occur, but not so quickly as to imply a medium/long-term dispersion of misalignments between countries. As shown below, this is the case for the BRICS+. Beta-convergence does not necessarily imply sigma-convergence.

Equation (6) is used to estimate the beta-convergence coefficient (by adding an error term and assuming that σ_ε^2 is constant and small, which is equivalent to assuming, for sake of simplicity, that shocks other than those of the real exchange rate determinants have little effect on countries' misalignment differentials). Then, we need to check whether or not the variance follows an ergodic process.

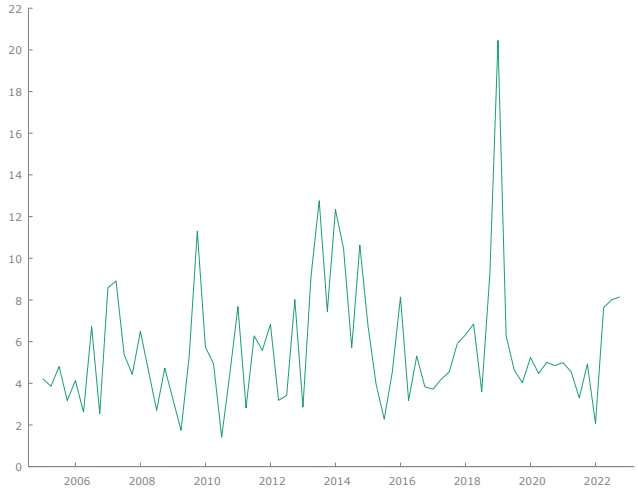


Figure 2. Variance of misalignment gaps

Figure 2 shows a graph of the temporal evolution of misalignment variance. The figure suggests a temporal instability of the variance over the quarters. To account for this, we give a time-varying estimate of the autoregressive coefficient of Equation 6. We use the kernel OLS time-varying estimator proposed by Giraitis et al. (2021). In Figure 3, we plot the squared of the coefficient β and its confidence interval. Therefore, a beta-convergence happens with a coefficient β varying between 0.8 and 0.9. The coefficient, close to 1, suggests a slow convergence. What is the implication for sigma-convergence?

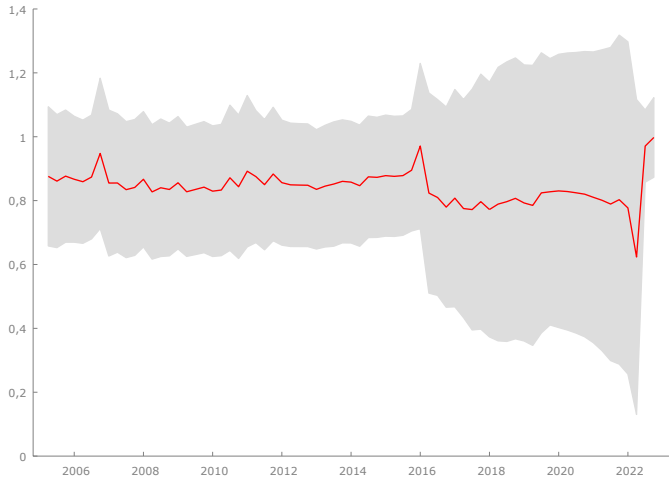


Figure 3. Time-varying estimation of the autoregressive term in Equation (6)

There are several types of ergodicity tests in the statistical literature. Here, we choose a visual approach that is simple to understand. We study the temporal memory of the dispersion of misalignments.

Our approach is based on the characterization of the memory of a time series. Consider a given time series $(X_t), t = 1, \ldots, T$ and define the vector of the partial means as $(\bar{M}_\tau), \tau = 1, \ldots, T$, where $\bar{M}_\tau = \sum_{t=1}^{\tau} X_t$. The process is ergodic is the variance of $\bar{M}_\tau$, defined by $S_\tau = var[\bar{M}_\tau]$, is bounded, i.e. $\lim_{\tau \to +\infty} S_\tau = C$, where C is a constant. This characterizes mean-ergodicity.

τ must be greater than T once we are able to generate obsversations of the series (X_t), beyond T. Successive observations of $\bar{M}_\tau$ up to $\tau = T$ serve as training data for models

predicting its evolution for $\tau = T$. We proceed as follows. The series (S_τ), initially comprises 71 observations (because we remove the first observation of $\bar{M}_\tau$ for which the variance is zero). We simulate between 500 and 1000 additional observations using a forecasting model. Figure 4a shows a selection of the forecasts obtained from a Generalized Regression Neural Network (GRNN) model. We select, optimally, 9 hidden nodes, 20 repetitions and retain a 6 autoregressive lagd. The reason for taking such a model is that the process underlying the series of misalignment dispersion is unknown, and potentially non-linear and complex. We find that the series (S_τ) diverges. The importance of being agnostic about the choice of forecasting model becomes apparent when we look at Figure 4b, which shows the evolution of S_τ when a linear model is chosen (here, based on AIC criteria, an ARMA(2,1)). We visually observe a convergence of the variance around 0.2. We may be wrong, however, given Figure 3, which seems to suggest that the temporal dispersion of misalignment gaps is not stationary.

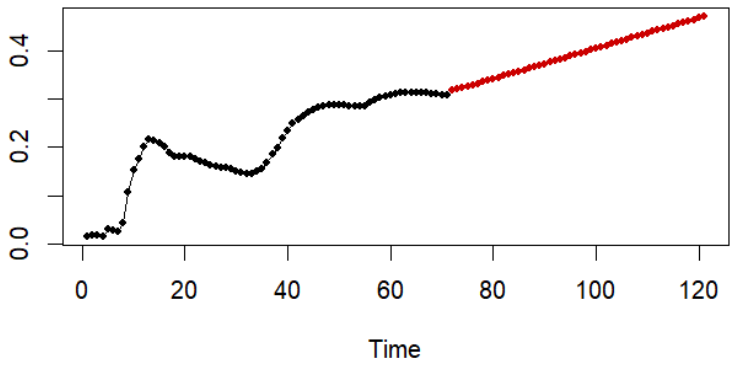


Figure 4a. Variance of partial average (from obs=72, the series is simulated using a general neural network model)

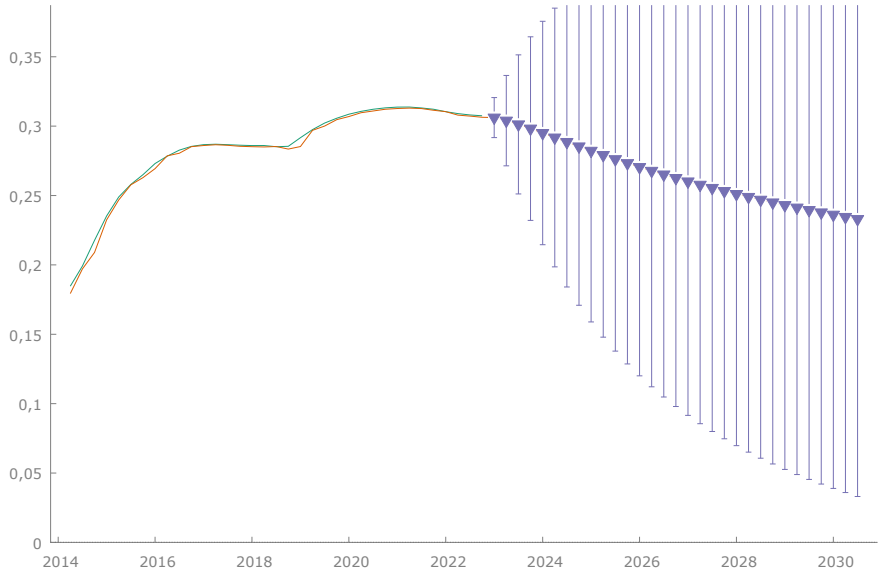


Figure 4b. Variance of partial average (from obs=72, the series is simulated using an ARMA(1,1) model). The bar represent the 95% confidence intervalley of forecast

More formally, since the coefficient β obtained by estimating Equation 6 is close to 1, the process underlying the dynamics of σ_{spt}^2 is described by a fractional long-memory model with a fractional parameter that we find to be equal to 0.56 with a standard deviation of 0.15 using the maximum likelihood method. The underlying process of σ_{spt}^2 can be written as :

$$(1-L)^d \sigma_{spt}^2 = \vartheta_t,\ \vartheta_t \sim iid,\ d = 0.56. \quad . \tag{8}$$

L is the lag operator defined by $L^i X_t = X_{t-i}$. It can be shown that for such a process, the rate of growth of the partial sums variances is such that $S_\tau = O\left(\frac{1}{T^{1-2d}}\right)$, which implies that $\lim_{\tau \to +\infty} S_\tau = \infty$. This a non-stationary long-memory model with a rate od decay of the memory described by a power-law function of order $2d$.

Our conclusion is that, if we adopt the convergence criteria on which our first definition of sustainability is based, the BRICS+ would not have constituted a sustainable economic, financial and monetary zone had the group been formed in the early 2000s. This would probably have been due to the difficulty of correcting their exchange rate misalignments by bringing them into line with those of the other member countries.

3.2.- Some evidence of clubs of convergence

We interpret the previous results suggesting that there is no evidence of any sustainability of a BRICS+ zone based on a joint convergence of their misalignments as a consequence of several convergence regimes between countries and the existence of "convergence clubs" in the misalignment dynamics.

To see this, assume a situation where countries decide on a common target for their real exchange rate, which is not their own equilibrium exchange rate, but the average equilibrium rate for the BRICS+ as a whole. The observed real exchange rate is allowed to deviate from this common target by a maximum percentage decided by the countries. The margin of tolerance is captured here by κ times the average dispersion of the equilibrium rate of all countries, where κ is a real number agreed upon by all countries.

Formally, we define a dummy variable for country i at time t :

$$S_{it} = \begin{cases} 1, & if\ \mu_t - \kappa\sigma_t < e_{it} < \mu_t + \kappa\sigma_t \\ 0, & Otherwise \end{cases} \quad (9)$$

μ_t is a measure of *the BRICS+' average* equilibrium exchange rates at time t, and σ_t is the corresponding standard deviation at time t (average and standard deviations can be simple or weighted). A country's misalignment thus corresponds to the following difference $e_{it} - \left(\frac{1}{10}\sum_{i=1}^{10}\gamma_i EE_{it}\right)$, where EE_{it} is the equilibrium exchange rate, of country i at time t, computed from the BEER model and the weight is defined by, either $\gamma_i = 1$ (simple average), or by the following number in case of weighted average: $\gamma_i = (t_i)^{1/3}\left(\frac{GDP_i}{\sum_{i=1}^{10} GDP_i}\right)^{1/3}(res_i)^{1/3}$. We take into consideration each country trade share t_i, GDP share and the amount of reserves as share of GDP, res_i.

From Equation (9) we define a vector of zeros and ones, which reflect two situations of, respectively, excessive misalignment and no excessive misalignment depending upon whether this difference lies inside or outside the interval $[-\kappa\sigma_t, \kappa\sigma_t]$.

Based on this framework, suppose that we assume that a common target zone for the BRIC+ is viable if the following condition holds:

Condition 1. Defining a common target for exchange rate misalignments is viable for the *BRICS+ if, on average over 3 consecutive years in the past (which could correspond, for*

example, to a period over which peer review would be exercised within the BRICS) a majority of countries do not experienced a situation of excessive misalignment.

Let us consider an example (Table 3). Suppose that, *for a given year*, we have the following data.

Table 3. An illustration of Condition 1

	Argentina	Brazil	China	India	South Africa	Saudi Arabia
S_{it}	0	0	1	0	0	1
γ_i	0.05	0.20	0.3	0.15	0.05	0.25

In this example, zeros indicate a situation of excessive misalignment based on Equation (9). Only two countries with important financial and economic weight within the BRICS+ are not in a situation of excessive misalignment (China and Saudi Arabia, i.e. 1/3 of the 6 countries, less than half). According to Condition 1, we would conclude against the assumption of viability of the group, by considering that the majority of more than 50% of the number of countries is not reached.

However, if we calculate the weighted average, we obtain 55%, which partly meets the condition we are proposing (only partly, as an average greater than or equal to 0.5 must be obtained for the average *of three consecutive years*, and not a given year). We therefore take account of the following 4 cases (see Table 4):

Table 4. Four ways of computing the percentage of countries with excessive misalignments

		Computation of μ_t in Equation 9	
		γ_i=1	γ_i: weighted average
Computation of the % of countries in excessive misalignments	Without γ_i	Case 1	Case 3
	With γ_i	Case2	Case 4

We will now show that, despite what one might think, the criterion of condition 1 does not prevent countries' misalignments from diverging from each other.

Figure 5a illustrates the evolution of the criteria used in definition 2 in the 4 cases, when $\kappa = 1$. The 0.5 bar represents the threshold of a majority of countries below excessive

misalignment. The x-axis shows the 3 consecutive years for which excessive misalignments are calculated (with sliding quarters). As can be seen, according to Condition 1, the BRICS+ group could have been considered as an economically and financially viable zone from 2014 onwards. We even see that the proportion of countries contributing to sustainability would have increased from 2018 onwards.

Figure 5b shows the evolution of sustainability indicator when $\kappa = 2$. The latter always lies above 0.5. If we chose a small value, for example 0.2, we would probably never conclude in favor of the viability hypothesis. Obviously, κ is a strategic parameter, since it means that governments need to agree on a value. It is a criterion that can be manipulated and depends on the political and economic balance of power within the BRICS+.

In fact, it would have been difficult for countries to agree on a value of κ. To see this, we consider an example corresponding to cases 3 or 4, with κ =1. We show the misalignments and the interval $[-\sigma_t, \sigma_t]$. (Figure 6). We can see that a group made up of the BRICS countries prior to the new members joining in 2024 (except China), would benefit from choosing a small value of κ, as their misalignments converge, remain relatively stable over time and evolve within an interval of a small length. Greater tolerance on misalignment bounds would have little influence on their trajectory (Brazil, Russia, India, South Africa).

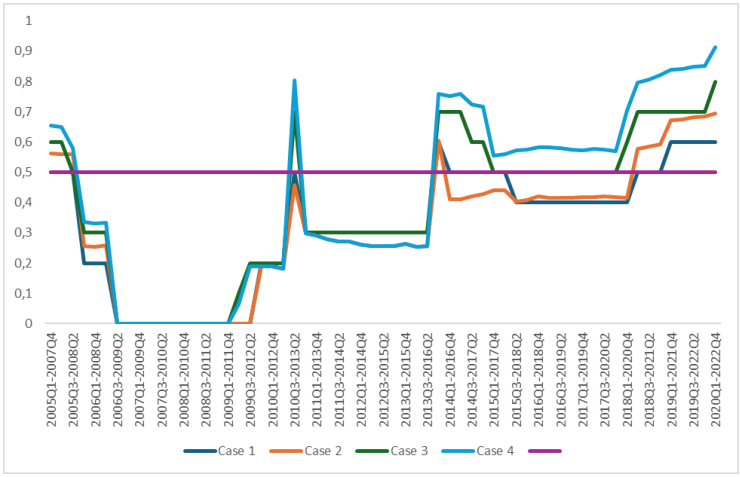


Figure 5a. Sustainability according to Definition 2 when $\kappa = 1$

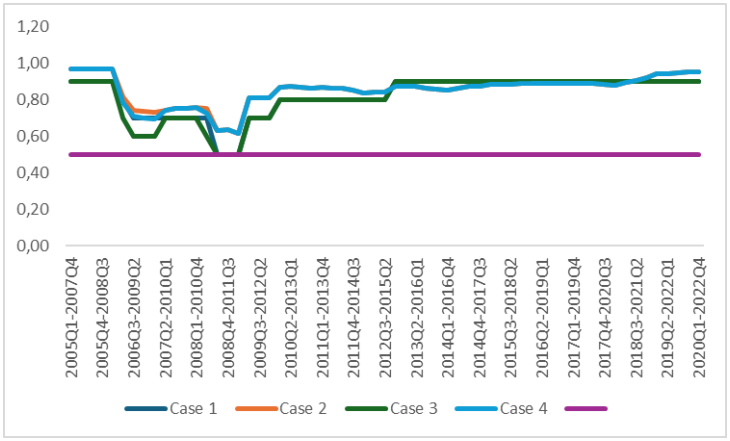
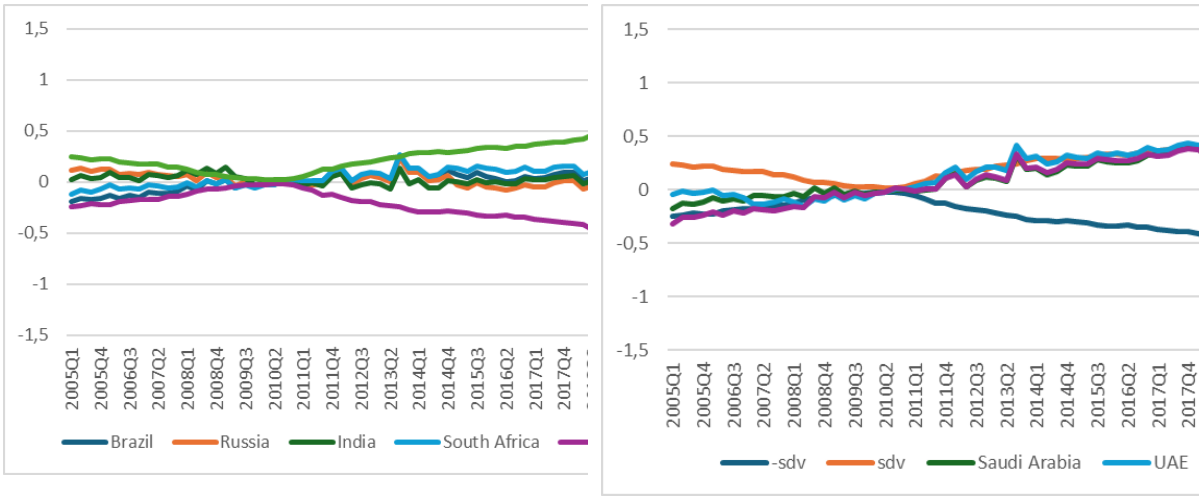


Figure 5b. Sustainability according to Definition 2 when $\kappa = 2$

We see two other groups of countries, which a small value of κ would penalize if they decided to restrict their misalignment to the same range as the previous countries. These are countries with higher fundamental real exchange rates whose misalignments follow the upper range of the interval (United Arab Emirates, Saudi Arabia and China).

Opposite this latter group, we have a third group with fundamental real exchange rates lower than that observed in the countries in the two former groups, and whose misalignments follow the lower bound, even exceeding it (Egypt, Ethiopia and Iran).

This illustrates what might be termed "convergence clubs" in the misalignment dynamics among BRICS+ countries.



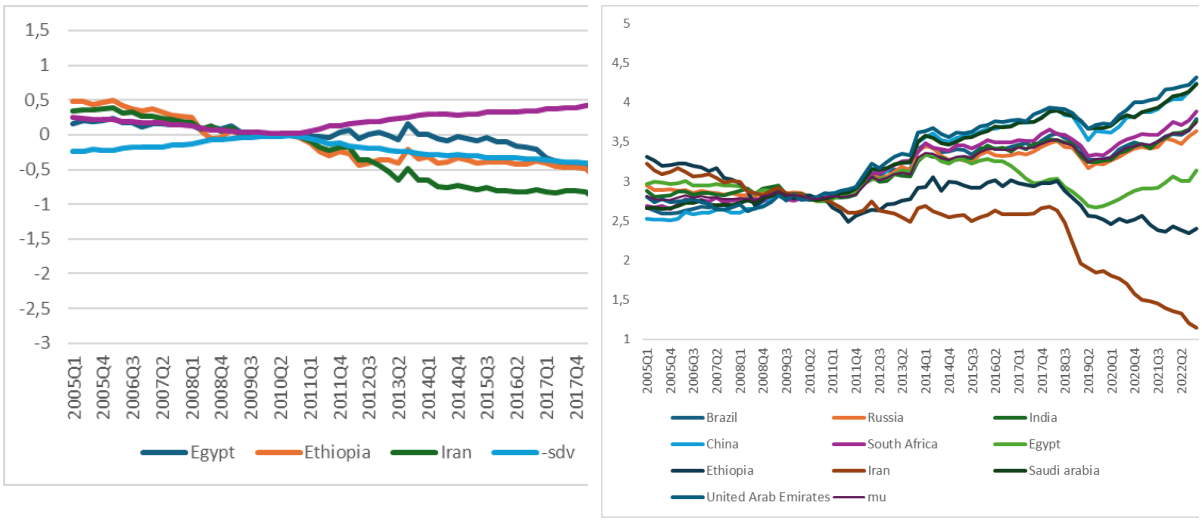


Figure 6. The dynamics of the misalignment ($e_{it} - \mu_t$) and the bounds $-\sigma_t$ and σ_t (first 3 figures). Countries' and weighted averaged BEER (fourth figure)

The last figure in Figure 6 can help us to understand one of the reasons why countries would have found it difficult for their misalignments to converge. We see that the equilibrium real exchange rates of each country diverge from one another. In particular, there are differences in levels. Brazil, Russia, India and South Africa have equilibrium exchange rates close to the weighted average equilibrium rate μ_t. Iran, Ethiopia and Egypt, from 2012 onwards, would have had a lower equilibrium exchange rate than the other countries, which means that their currencies would have been overvalued relative to the others'. And over time, this over-valuation would have increased. In contrast, the United Arab Emirates, China and Saudi Arabia would have had undervalued currencies compared to the other countries. However, the gap between their BEER trajectories and those of the other original BRICS weaker. These observations mean that the costs in terms of exchange rate adjustment (and therefore macroeconomic and financial policies) of joining the member other countries' misalignments would have been very high for Iran, Ethiopia and Egypt. These three countries therefore appear as outsiders. When they are not considered in the sample, the misalignment divergence is much more attenuated, as shown in Figure 7. The new variance of misalignments leads to a "channel" whose width remains almost constant over time.

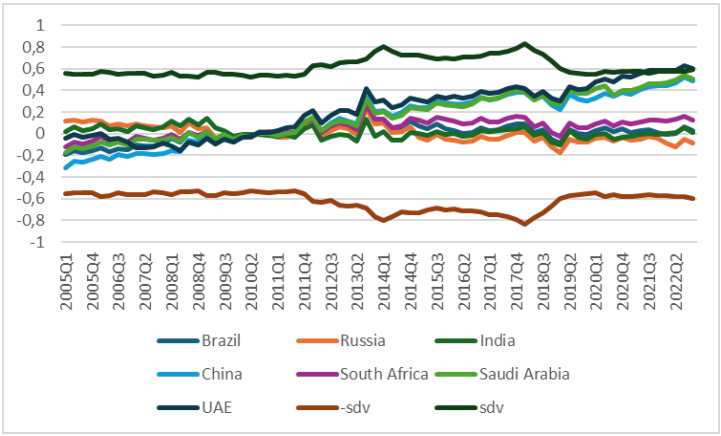


Figure 7. The dynamics of the misalignment ($e_{it} - \mu_t$) and the bounds $-\sigma_t$ and σ_t without Iran, Egypt and Ethiopia

Figure 6 and 7 show that the convergence of misalignments between the BRICS+ would have gone through three phases. A phase of convergence of misalignment trajectories from 2005 to 2008, a phase during which all countries would have had very close misalignments, from 2009 to 2011. Finally, from 2012 onwards, the trajectories would have diverged between different groups of countries.

The "butterfly" dynamics is linked to the price differential between countries (measured in Figure 8 in log). The countries went through three phases: an initial phase of price convergence, a phase corresponding to two years with no significant difference in prices, and finally a phase of price divergence. The common nominal exchange rate against the dollar is stable until 2011, then again from 2019 onwards. It depreciates between 2012 and 2017, then appreciates briefly in 2018.

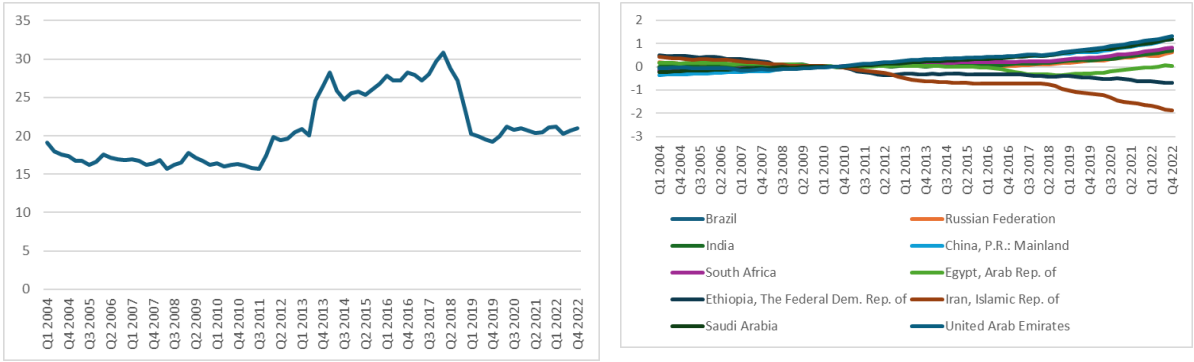


Figure 8. Exchange rates: Common nominal exchange rates s^{brics} (left) and log of price differentials (right)

The previous results show two configurations. On the one hand, some of the countries have lower real interest rates than the BRICS+ average, which should penalize capital inflows and lead to a relative depreciation of the currency against other currencies for instance, Iran). By forcing them to adopt the same common exchange rate, this leads to an over-appreciation of their real exchange rate (e_{it} is lower). The same argument applies to Ethiopia and Egypt, which have lower foreign exchange reserves than the other countries. Conversely, countries such as the United Arab Emirates, China and Saudi Arabia have accumulated high levels of foreign exchange reserves. This should lead to an appreciation of their currencies. But, by adopting the nominal exchange rate common to all BRICS+, the real exchange rate is undervalued relative to the level at which it should be.

Do these results make the BRICS+ de-dollarization project unfeasible? The divergence is primarily due to a difference in relative prices. We could imagine that, to mitigate the effects of this divergence on the real exchange rate, a monetary entity common to the countries would decide not to let the currency fluctuate against the dollar, but would intervene on the foreign exchange markets by, for example, engaging in a policy of managed floating.

4.-Conclusion

The case of the BRICS illustrates a classic situation of countries that have structural divergences and macroeconomic fundamentals, but which, if they decide to adopt the same currency - which will be the case of the BRICS+ if they want an alternative currency to the dollar - may encounter difficulties in correcting their imbalances. If this common currency is defined according to the weight of the countries, then it could be too strong a currency for countries like Iran, Egypt and Ethiopia, and on the contrary, it could benefit countries like the UAE, Saudi Arabia or China by being an undervalued currency in relation to their macroeconomic and financial fundamentals.

As a first step, countries could agree to converge on a few key variables, such as interest rates, by developing a common BRICS+ financial market through greater financial integration. This is possible if there is greater commercial integration between them, which could be encouraged by the adoption of the same invoicing currencies (their currencies or at least the Renminbi). This convergence of interest rates could also be achieved through a common debt market - denominated in one of the countries' currencies - notably to finance the development of the zone's least developed countries (Egypt, South Africa, Ethiopia, Iran, Russia...). Indeed,

this market-based solution seems more realistic than a strategy of transferring structural funds, as in the case of the euro zone.

The aim of our work has been to show that building an economic and financial union to support a common currency is unsustainable if structural divergences between countries are too strong (as illustrated in the case of the BRICS+ by a divergence in price levels). Countries would only have an incentive to deviate from the common currency, since by adopting a currency different from the others, they could hope to adjust the nominal exchange rate to compensate for the effects of price differences. So, this leads to an interesting question about the optimal exchange rate policy for a BRICS+ zone. Rather than letting the common currency fluctuate against the dollar according to the evolution of countries' respective exchange rates against the dollar, we could ask ourselves, at what level should the common exchange rate be set to reduce or even eliminate the differences observed between real exchange rates?

The experience of the euro zone shows that when a group of countries chooses a common currency with a floating exchange rate, macroeconomic adjustment relies exclusively on government efforts to reduce macro-financial imbalances and correct structural imbalances. Their experience also shows that, when they fail to do so, there is a risk of even greater divisions, which may lead countries to adopt policies of fiscal austerity to avoid calling the single currency into question. The BRICS+ group is made up of developing and emerging countries. It is unthinkable to leave the exclusive responsibility for adjusting to shocks that would cause economies to diverge to countries' macroeconomic, financial and structural policies. In this context, nominal exchange rate management policies are more appropriate.

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Appendix : Data, sources and description

This appendix describes the data and sources.

Relative productivity

Relative productivity is computed as the ratio of each country productivity and the weighted average of the productivities of the other BRICS member. Weights are computed by the share of a country's trade with each of the other BRICS members, relative to total trade with all members.

Log difference =LN(country productivity)－LN(BRICs average productivity)

Source: pwt 10.01. Series ctfp (TFP level at current PPPs – USA=1).

Frequency = annual from 2004 to 2019

Data are missing for UAE and Ethiopia. For these countries, we assume that their TFP = average TFP of the other member countries. So relative productivity equals LN(1)=0

Bilateral trade flows

Trade share of bilateral trade flows. More specifically the share of a country's trade with the other BRICS members, relative to total trade between all members. Trade is measured by the sum of bilateral exports and imports.

Source: IMF Direction of Trade Statistics.

Imports, US dollars Millions; Exports, US dollars Millions FOB

Frequency=Quarterly data from 2004q1 to 2022q4.

Terms of trade

Net barter terms of trade index (2015 = 100) is calculated as the percentage ratio of the export unit value indexes to the import unit value indexes.

Terms of trade defined as the ratio of exports and import prices (in log).

Source: World Development indicators

Frequency = annual from 2004 to2021.

Inflation differentials

Difference between a country's inflation and the weighted average inflation of the other BRICS members. The weights are calculated taking account of the share of each country GDP over total GDPs of the BRICS members.

Source: World Bank. Consumer Price Index, All items, Percentage change, Previous period.

Frequency = Quarterly data from 2004q1 to 2022q4. All countries except Egypt and United Arab Emirates.

Egypt: Core inflation from Central Bank of Egypt. Original data monthly, converted to quarterly.

UAE: source Fred Saint-Louis, Annual converted to quarterly by interpolation.

Interest rate differentials

Interest rate differences between domestic interest rate on Treasury securities and the average of BRICS members' real long-term interest rates.

Real interest rate= nominal rate – inflation rate

India, Russia, Brazil, South Africa: Interest Rates: Long-Term Government Bond Yields: Frequency=10 Year: Source: OECD. Quarterly. For Russia available until 2018q2. Completed using IMF data

China: 3-Month or 90-Day Rates and Yields: Treasury Securities. Source: FRED Saint-Louis. Original: monthly converted to quarterly. OECD quarterly database available starting in 2013. The two interest rates (10 year and 3-months evolve jointly both in level and trend).

Egypt. Source IMF. Financial, Interest Rates, Government Securities, Treasury Bills Since 2017 available quarterly. Between 2004-2016 available quarterly. We interpolate to obtain quarterly data.

The weighted average of BRICS members' real long-term interest rates (nominal deflated with inflation rate). The weight is calculated based on each country's sovereign debt rating. Our idea here is that the better the quality of a country's debt, the more likely is this country to become a benchmark country for foreign investors, as a basis for defining the spreads of the other countries. Such a variable captures the role of financial flows between the BRICS countries.

The following number of sovereign debt rating is based on the S&P rating (cell No. P85~)

AAA+100 AAA 98 AAA- 96 AA+ 95 AA 90 AA- 85 A+ 80 A 75 A- 70

BBB+ 65 BBB 60 BBB- 55 BB+ 50 BB 45 BB- 40 B+ 35 B 30 B- 25

CCC+ 20 CCC 15 CCC- 10 CC+ CC 8 CC- C+ C 5 C- D~0

The weights are calculated taking account of the share of each country sovereign debt rating over total sovereign debt ratings of the BRICS members.

Reserve assets to nominal GDP

Reserves = Gross International Reserves

We compute the ratio of a country's reserve assets to nominal GDP (in log)

Source : FRED Database for the following countries

Monthly data from 2004m1 to 2022 m12, converted to quarterly : Brazil, China, India, Russia, South Africa; total reserves excluding gold in dollars.

Source: World Development Indicators

Annual data from 2004 to 2022 converted to quarterly : Saudia Arabia, AUE, Egypt, Iran, Ethiopia

Debt ratio as share of GDP

Debt = General government gross debt

Debt ratio as share of GDP (in log)

Source : FRED Database.

Frequency : annual from 2004 to 2022; converted to quarterly

Computation of the bilateral nominal exchange rate s^{brics}.

As explained in Section 3.1, we compute the bilateral nominal exchange rate according to Equation (3) :

$$s^{brics} = \prod_{k=1}^{K} (s^k)^{\theta_k},\ \theta_1 + \theta_2 + \ldots + \theta_K = 1,\ 0 \leq \theta_k \leq 1.$$

s^k is the exchange rate of the currency of country k against the dollar, θ_k is the weight of country k is the basket computed by considering bilateral trade share, foreign reserves shares and GDP shares. To compute the weights, we consider the sum of each country's share of bilateral trade with the other BRICS+ members, GDP share in total of the BRICS+'s GDP, and each country foreign reserves in the total of the BRICS+'s foreign reserves. (see Appendix 2 for a detailed description of the computation of the weights).

Computation of trade share

$$t^k = \frac{(X^{k,brics} + M^{k,brics})/GDP^k}{\sum_{k=1}^{K}\{\frac{(X^{k,brics} + M^{k,brics})}{GDP^k}\}}$$

t^k is country k's bilateral trade share, $X^{k,brics}$ and $M^{k,brics}$ are respectively country k's exports (resp. imports) to (resp. from) the other BRICS members. As mentioned before, our endogenous variable is a country's bilateral real exchange rate vis-à-vis a numeraire, which we construct as a "multilateral" currency of the BRICS members. GDP^k is country k's GDP.
Data on bilateral exports and imports are taken from BACI database from CEPII.

Computation of the shares of foreign reserves

For each country *k*, we compute the share of reserves as ratio of broad money, short-term debt and imports, each divided by the average of the BRICS members (including country k). We call these share $res\_mon^k, res\_debt^k, res\_imp^k$. From these ratios, we take the geometric average: reserve/ broad money, reserve/debt, reserve/imports

$$res\_mon^k = \frac{(reserve^k)/(broad\ money^k)}{\left\{\sum_{k=1}^{K}\left(\frac{reserve^k}{broad\ money^k}\right)\right\}/K}$$

We use short-term debt (% of total reserves) taken from World development indicators. Short-term debt includes all debt having an original maturity of one year or less and interest in arrears on long-term debt. Data missing for Saudi Arabia and UAE. For these countries, we use the external debt (short- and long-terms).

$$res^k = (res\_mon^k)^{1/3}(res\_debt^k)^{1/3}(res\_imp^k)^{1/3}$$

Finally, we construct the following index for a country k

$$I^k = (t^k)^{1/3}\left(\frac{GDP}{Total\ GDP}^k\right)^{1/3}(res^k)^{1/3}$$

The index I^k is rescaled to vary between 0 and 1 (that is θ_k) and so that their sum equal 1. We should note that I^k is normally time-varying (one index per year or month).

We assume fixed weights by computing time average.