Higher Education Institutions as multi-product services exporters

Evidence from the UK

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1 Introduction

Universities as multi-product service exporters

- Focusing on teaching, variety over disicplines and levels
- Increasing share of foreign students: mode 2 service exports
- Equivalence to multi-product goods exporters
  - market structure and regulation
  - consumption and preferences
- ...

How does foreign demand affect universities' programme (product) mix?

- Foreign demand ↑, especially for English language education
- In multi-product firms opening to foreign trade:
  - export their most profitable products (Bernard, Redding & Schot, 2011)
  - choice to export may be affected by quality and product ladder (Qiu & Zhou, 2013; Manova and Yu, 2017)
- Universities
  - Multi-line (multiple subject areas) - multi-product (degrees within) (or should we see degrees as an intensive margin?)
  - Would exports drive an increase in the intensive and extensive margin along a product ladder?
  - Implies a concentration of their offer along core competences
  - But... Universities are different from profit maximising firms (non-profit, price caps, subsided, active beyond teaching (Johnes, 1993, 2012))

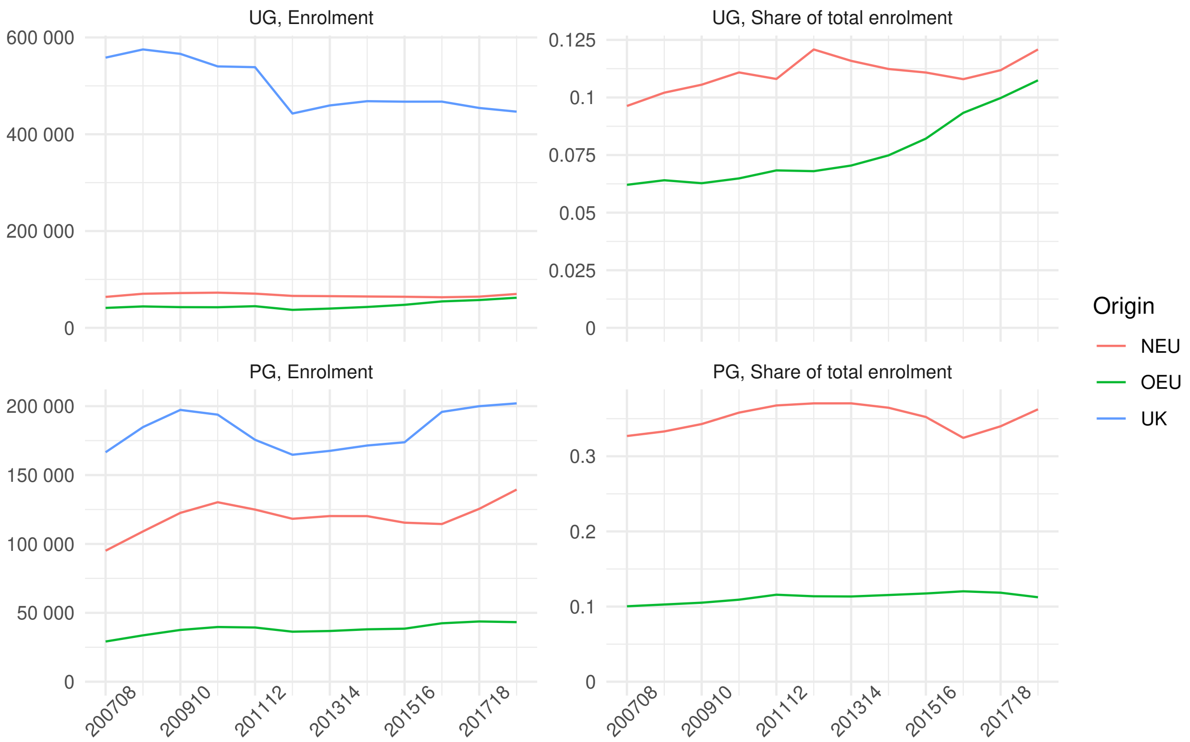
Is it big?

In the UK, according to WTO Trade in Services data by mode of supply

- US$ 9.9 billion
- 19% of all M2 exports,
- 1.5% of all services trade, 2.5% when finance excl.

  +

- Local spending on food, accommodation, recreation...



Literature on services exports using micro-data

- Services trade less studied than goods trade, due to lack of data (not covered by the same registration requirements as goods trade).
- Country studies on incidence of services exports/imports from Belgium (Ariu, 2016), France (Crozet et al, 2016), US (Tito 2019), Italy (Bamieh et al., 2021), Portugal (Amador 2019), 8 OECD countries (Rouzet et al., 2017, Benz et al, 2020).
- Extensive margins in terms of destinations (Christen et al. 2018)
- Service products exports among (large) manufacturing firms in Italy (Federico & Tosti 2017).

This paper: UK Universities (~100), from 2013/14 - 2018/19, 100+ products (principal subjects), across 2 levels of studies (UG and PG), with enrolment numbers for domestic and foreign students.

2 Data and Stylised facts

Data

First-year enrolment data from the Higher Education Statistical Agency (HESA)

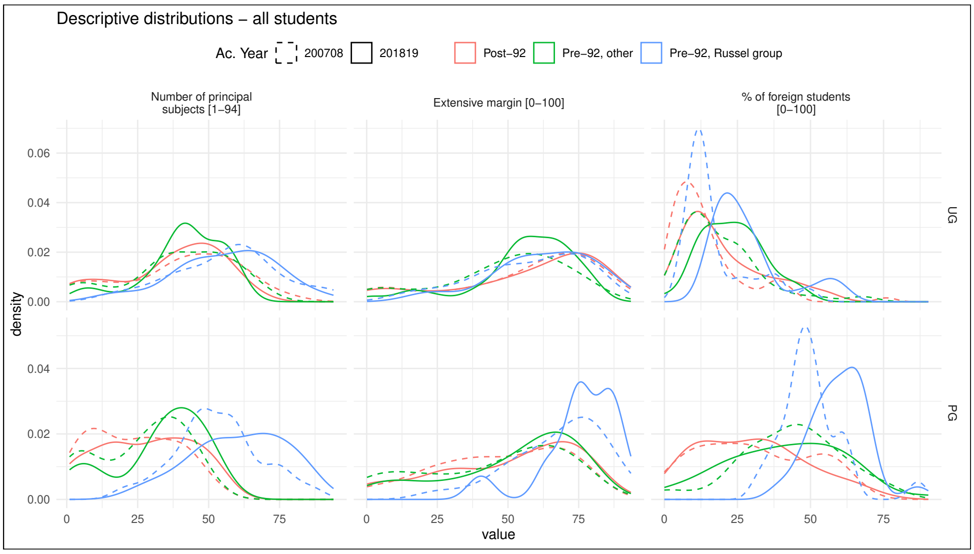
- Higher Education Institution (157), i.e. university
- Level of study, i.e. under-graduate and post-graduate
- Principal subject (170), e.g. economics, politics, civil engineering
  - These aggregate to 21 subject areas, e.g. social sciences, engineering.
- Academic years, from 2013/14 to 2018/19.
- For all analysis, only looking at first year of study.

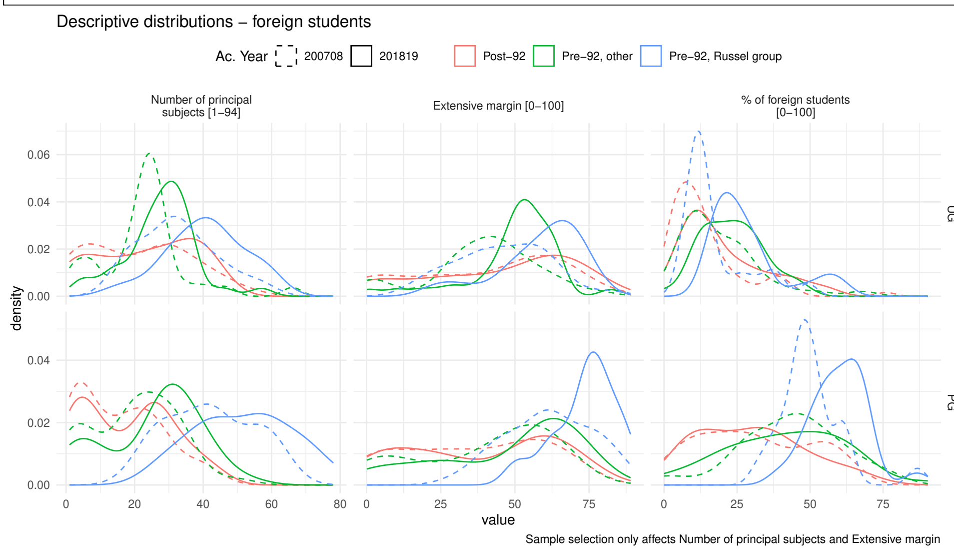
Stylised facts

Variety in

1. Range of principal subjects across universities
2. Share of foreign students across universities
3. Frequent entry and exit of subjects across universities

Subjects and foreign students across universities



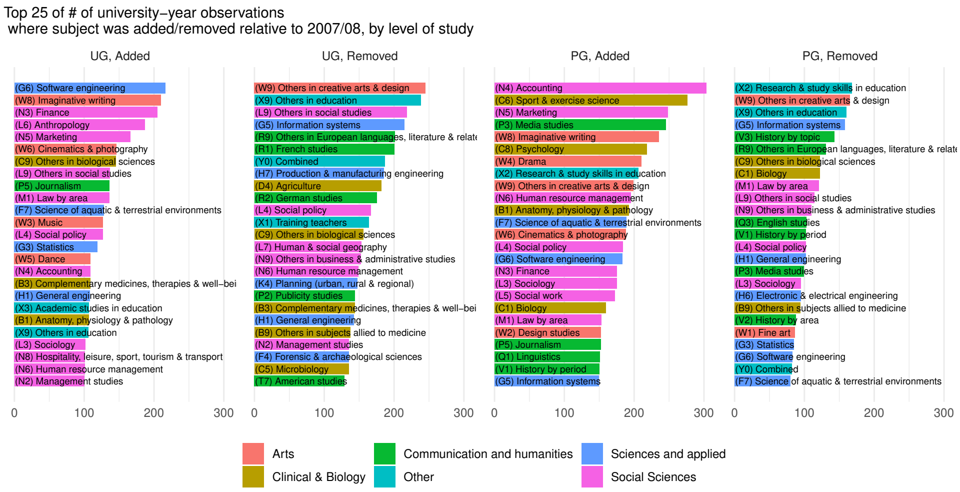


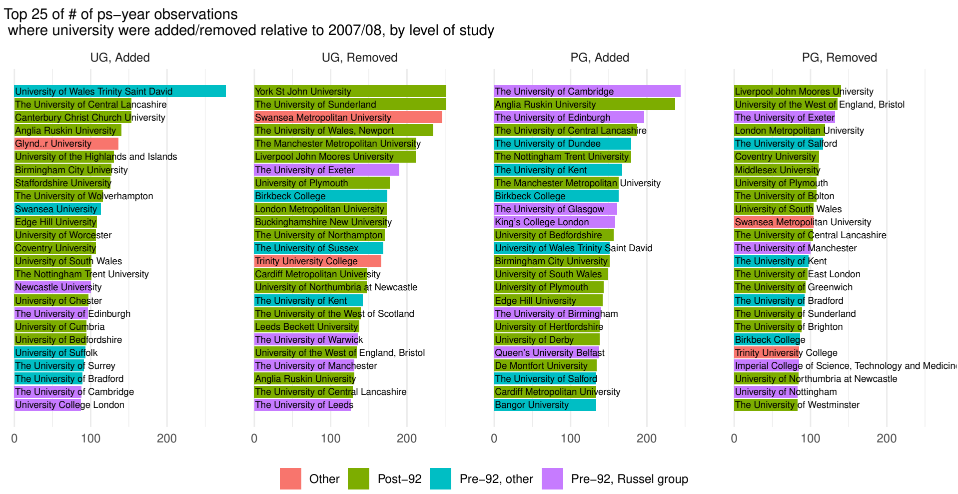
There appear two types of universities: those quite specialised: ~10 subjects, and those much more broader ~40 subjects (left panels). But when weighting n.ps against national student share there is less variation to get to a normalised extensive margin (middle panels) there seems to be more of a continuum, especially for UG. The foreign student share of PG is much higher than in UG. There is some variation over time, especially UG is getting more international across the board.

The three types of universities are noticibly different from each other. The Russel group universities (which combine the largest and older insitutions, often also more research intensive than the others) tend to offer more subjects, and a larger extensive margin and higher foreign student share. Even if they already were larger and more international at the beginning of the sample, the distributions clearly shifted to the right across the three indicators, increasing the extensive margin and the foreign student share for both UG and PG. Post-92 institutions are former polytechnical schools that were designated full universities in 1992. While offereing fewer subjects and a smaller foreign student share relative to the Russel group universities, the distribution across all three indicators has shifted to the left as well. The other pre-92 institutions tend to be a bit inbetween the Russel group and then the post-92 institutions.

Universities change their offer regularly

The distributions on the number of principal subjects offered by universities already suggest that univerisites change their offering over time. It can be shown more explicitly that universities' vary their subject offer regularly (identified by the appearance and disappearance of university-subject combinations by year, based on student enrolment larger than one). The following charts show which subjects are most frequent added or removed from the offer of universities, and which universities see most year-to-year changes in the subjects that they are offering.





3 A Model of University as service exporters

Model overview

The full model is presented formally in the appendix.

- 2-country model
- Demand:
  - consumers demand varieties across subjects, affected by price and taste
- Supply:
  - Fixed number of institutions, fixed number of global principal subjects. The interaction gives varieties.
  - Universities produce across multiple subjects, subject to fixed and variable costs. Fixed costs vary by student origin.
  - Varieties are university-principal subjects, indexed by the productivitity draw.

Demand for education

Student utility

$$U_t = \frac{1}{\gamma} C_t^{\gamma},\ C_t = \sum^{J} \left(\frac{C_{i,t}}{\alpha_i} \right)^{\alpha_i}$$

over J subjects, with preference weigth α_i for a subject.
Within each subject there are different varieties: universities offering programmes in subject J.
Students can follow domestic or foreign courses

$$C_{i,t} = \left(\int_{\omega \in \Omega_t} \left(\lambda_i \left(\omega \right) c_{Di,t} \left(\omega \right) \right)^{1 - \frac{1}{\sigma_i}} d\omega + \int_{\omega^* \in \Omega_t} \left(\lambda_i^* \left(\omega^* \right) c_{Xi,t} \left(\omega^* \right) \right)^{1 - \frac{1}{\sigma_i}} d\omega^* \right)^{\frac{1}{1 - \frac{1}{\sigma_i}}}$$

$\lambda_i(\omega)$ and $\lambda_i^*(\omega)$ are taste/quality shifters for variety ω, both for domestic and foreign.

This leads to the usual expressions on demand for a variety, summarised thorugh a price index, that accounts for the subject preferences and the taste/quality shifter.

Supply of education

N,N^* Universities in each country, exogenously defined.
Each university draws

- Productivity level, φ from $G(\varphi)$, with support on $[\varphi_{\min}, \infty)$.
- Taste level λ for a subject i, from $Z(\lambda_i)$, with support on $[\lambda_{i\,\min}, \infty)$

Production requires a variable cost, w_i/φ, and a fixed costs that are subject and market specific: accepting foreign students implies a higher fixed costs ($f_{X,i} > f_{D,i}$).

Solving

- The representative student maximise utility given preferences across varieties, tastes for specific topics at universities and prices.
- Universities *maximise profits*, given variable and fixed costs and demand.
- The productivity of an institution interacts with the subject specific tastes and fixed costs to create a selection mechanisms where some unverisities do not offer a certain subject at all, or not to foreign students.

Model overview

What determines the number of principal subjects that are offered in each university?

$$\underbrace{\frac{M_{Xi,t}\widetilde{\rho}_{Xi,t}\widetilde{y}_{Xi.t}}{N_t}}_{\text{margin}} = \underbrace{\Delta\left(\frac{\rho^*_{i,t}}{\tau}\right)^{v}\left(\frac{Q_t}{w_t}\right)^{v\frac{\sigma_i}{\sigma_i-1}-1}}_{\text{national time varying}} \times \underbrace{(\alpha_i C^*_t)^{\frac{v}{\sigma_i-1}}}_{\text{for. demand is uni spec}} \times \underbrace{f_{Xi,t}^{-\left(\frac{v}{\sigma_i-1}-1\right)}}_{\text{uni specific aspects}}$$

4 Empirics

Empirical model

We are interested in estimating the following equation, that directly relates to the 'gravity'-type equation resulting from the theory. See the appendix for the full derivation.

$$m_{i,t} = \beta \underbrace{Y_t^{\text{Asian}}}_{\text{foreign demand}} \times \underbrace{w_i \times \text{group}_i}_{\text{uni specific impact}} + \mu_i + \theta_t + \varepsilon_{i,t}, \tag{1}$$

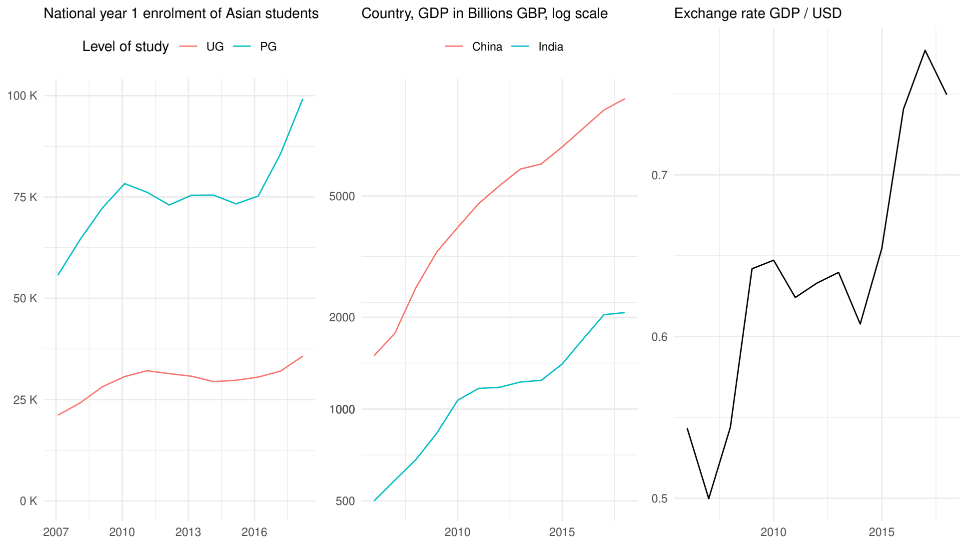
- $m_{i,t}$: a measure of the trade margins of universities.
- Y_t^{Asian}: time varying driver representing aggregate demand from Asian students (related to C_t in model).
- w_i: a dummy variable on university exposure.
- group_i: a variable to distinguish the effect across different types of universities.
- μ_i, θ_t: university and time fixed effects (all other factors that are common across universities and time).
- β is the estimate on the effect of universities that are relatively more affected by asian demand, separated by university type.

Model is separately estimated by level of study (UG, PG).

Defining treatment indicator, $w_i \times Y_t^{\textbf{Asian}}$

In order to estimate β, we need to differentiate between universities based on their exposure in an increase in foreign demand that is inherently common across all universities. We postulate that we can use the surge in demand from Asian students (India and China combined) is concentrated among specific subjects, and that some universities were more exposed because they already had a specific expertise in these specific subjects. Therefore, we need to define the following,

- Y_t^{Asian}: time variation
  - exogenous driver of surge in enrolment
- w_i: cross-section
  - Identifier of universities that are most likely to be affected by surge in exogenous demand for enrolment



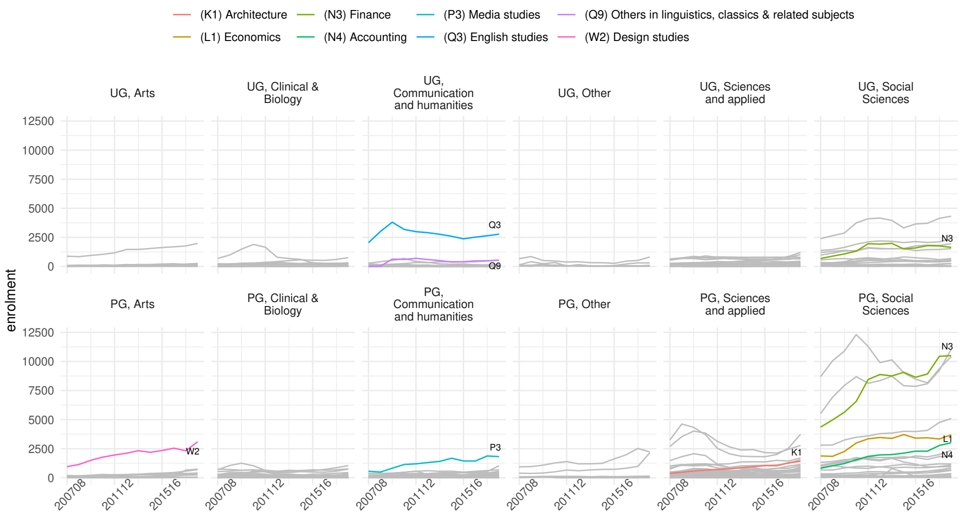
As enrolment numbers are endogenous (left panel), we require the exogenous driver of increased student demand. The rise in Asian enrolment can be directly related to the growth in GDP of India and China (panel 2). Especially when set in GBP, because the periods of depreciation and appreciation of the GBP against the USD (but also against other currencies, panel 3) is reflected in the country GDP statistics and the enrolment patterns for PG.

Where did all these students go? While Asian students are free to go to any subject, there is a clear concentration into some. For instance, some engineering and business related studies tend to see high enrolment and high shares within subject from Asian students. We use four criteria to identify the specific subjects in which Asian students were most likely to choose. Having identified the subjects, we look at which universities had a sort of "comparative advantage" in these subjects, measured as the overal enrolment share within the university. Hence,

1. Find subjects that experienced strong demand from Asia
2. Define treated universities as those that were strong in these subjects at start of period

Criteria of subjects most exposed to Asian enrolment	PG	UG
Asian student enrolment at least x during one year	x=1000	x=500
Percent of Asian students at least x during one year	x=25	x=15
Percentage point growth in Asian student enrolment over the entire period	10	10
Total Asian enrolment growth over the entire period	1000	500

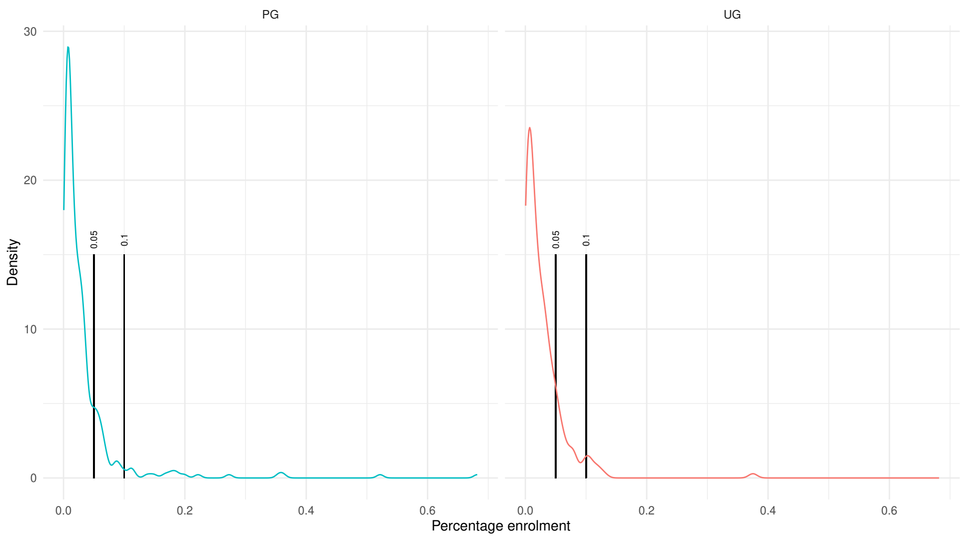
w_i,subjects that experienced a strong demand from Asia



Enrolment numbers of Asian students by principal subjects (PS) with the PS that are selected with the above criteria are highlighted in the chart. Some grey lines (not selected principal subjects) appear above the highlighted ones. This is indicates that they may satisfy the absolute enrolment criteria, but at least one of the other three criteria is not met.

More PG subjects are selected than UG subjects even though for UG we use less restrictive selection criteria. This demonstrates again the importance of Asian students for the PG level compared to the UG level in absolute and relative enrolment.

w_i,Universities that have large overal student shares in selected subjects



The figure shows the density of the share of enrolment of the selected principal subjects (so observations are university-principal subjects) for the start of the period. Observations that are to the right of the threshold lines will define which universities are considered as treated. Hence a university can be considered treated if it has at least one of the selected subjects with enrolment share above the chosen threshold.

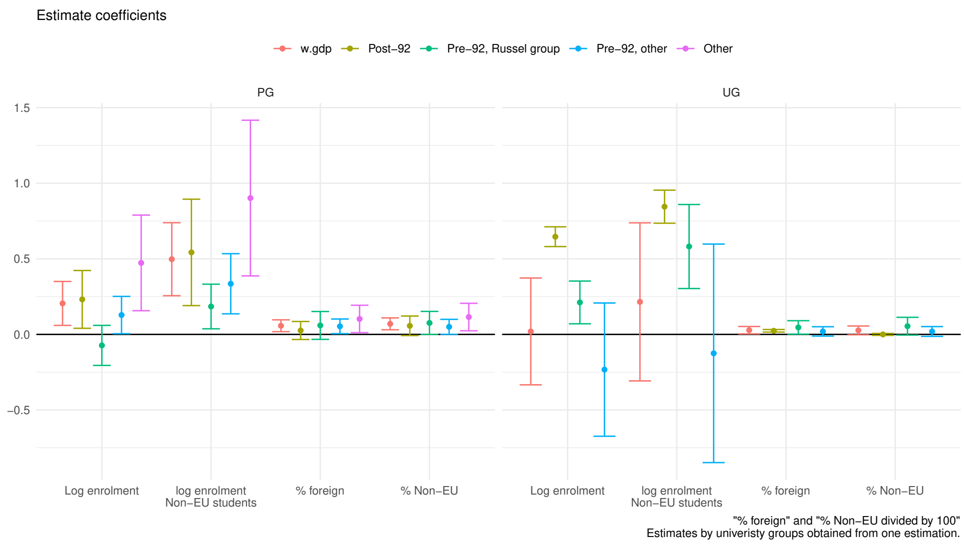
A threshold of 10% of a principal subjects specific enrolment share in total university's enrolment selects universities that are most concentrated in these PS. A lower threshold (5%) would still capture relative few institutions, but also lowers the potential effect from Asian enrolments because subjects may be quite small relative to total enrolment (i.e. around 5%).

The table shows the number of universities selected, by the threshold levels indicated above and seperated by type of university. In the final results we have used the 0.10 threshold.

	≥ 0.10		≥ 0.5	
University groups	UG	PG	UG	PG
Post-92	1	4	10	13
Pre-92, Russel group	2	2	7	10
Pre-92, other	4	5	13	15
Other	0	4	1	4

Results

Treatment effect on enrolment measures



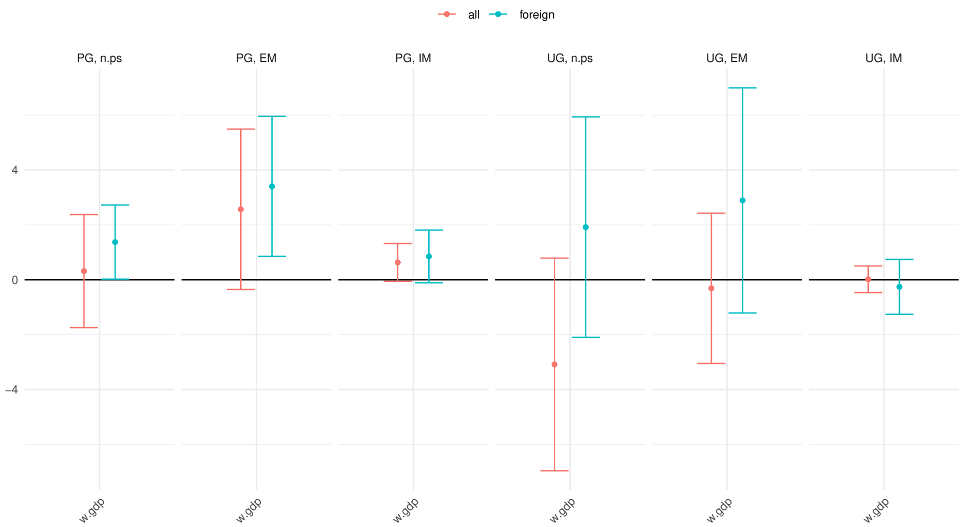
The graph presents the results, while the table provides additional details. For PG, enrolment and foreign enrolment increase more for universities that are concentrated in studies that are favoured by Asian students. The coefficient on *w.gdp* presents the total effect, while the other coefficients split it out across types of universities. Post-92 universities and pre-92 tend to have grown more substantially relative to the Russel group universities.

Treatment effect on trade margins

Table 1: Treatment effect on enrolment measures

	1. l.e	2. l.e	3. l.e.NEU	4. l.e.NEU	5. pct.e.NEU	6. pct.e.NEU	7. pct.e.for	8. pct.e.for
[0.5em] *UG*								
w.gdp	0.019		0.215		2.637+		2.776*	
	(0.180)		(0.267)		(1.482)		(1.241)	
w.gdp:Post-92		0.646***		0.845***		-0.023		2.369***
		(0.033)		(0.056)		(0.328)		(0.435)
w.gdp:Pre-92, other		-0.233		-0.125		1.914		1.966
		(0.225)		(0.369)		(1.651)		(1.579)
w.gdp:Pre-92, Russel group		0.211**		0.581***		5.412+		4.601*
		(0.072)		(0.142)		(3.002)		(2.271)
Num.Obs.	1879	1879	1879	1879	1879	1879	1879	1879
[0.5em] *PG*								
w.gdp	0.205**		0.497***		6.980***		5.703**	
	(0.074)		(0.123)		(2.006)		(1.998)	
w.gdp:Other		0.473**		0.902***		11.479*		10.216*
		(0.161)		(0.263)		(4.639)		(4.618)
w.gdp:Post-92		0.231*		0.542**		5.675+		2.562
		(0.097)		(0.180)		(3.296)		(3.040)
w.gdp:Pre-92, other		0.128*		0.335**		4.985+		5.322*
		(0.063)		(0.101)		(2.530)		(2.474)
w.gdp:Pre-92, Russel group		-0.073		0.184*		7.580+		5.919
		(0.068)		(0.075)		(3.887)		(4.694)
Num.Obs.	1944	1944	1944	1944	1944	1944	1944	1944

+ p < 0.1, * p < 0.05, ** p < 0.01, *** p < 0.001

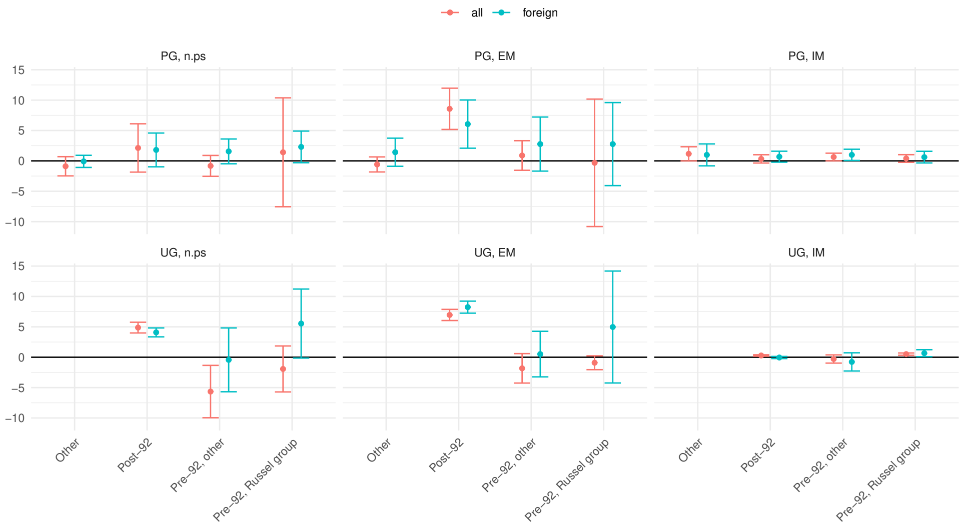


	1. EM	2. IM	3. n.ps	4. EM	5. IM	6. n.ps
[0.5em] *UG*						
w.gdp	-0.312	0.018	-3.086	2.892	-0.259	1.919
	(1.397)	(0.247)	(1.977)	(2.093)	(0.510)	(2.050)
Num.Obs.	1879	1879	1879	1866	1866	1866
[0.5em] *PG*						
w.gdp	2.569+	0.631+	0.317	3.404**	0.851+	1.373*
	(1.491)	(0.352)	(1.051)	(1.302)	(0.490)	(0.689)
Num.Obs.	1944	1944	1944	1908	1908	1908
sample	All	All	All	Foreign	Foreign	Foreign

+ p < 0.1, * p < 0.05, ** p < 0.01, *** p < 0.001

The average effect across all universities indicates that the Asian student surge had a most notable effect on the extensive exporting margins, suggesting an expansion of subjects across universities, especially into those that are important at the national level. A similar but slightly smaller effect is estimated for margins based on all students (domestic and foreing), but this effect is not statistically significant.

Treatment effect on trade margins by university grouping



	1. lhs: EM	2. lhs: IM	3. lhs: n.ps	4. lhs: EM	5. lhs: IM	6. lhs: n.ps
[0.5em] *UG*						
w.gdp:Post-92	6.955***	0.284***	4.875***	8.240***	-0.049	4.085***
	(0.467)	(0.059)	(0.450)	(0.504)	(0.088)	(0.374)
w.gdp:Pre-92, other	-1.830	-0.294	-5.653*	0.515	-0.772	-0.433
	(1.236)	(0.345)	(2.198)	(1.913)	(0.765)	(2.679)
w.gdp:Pre-92, Russel group	-0.909	0.510***	-1.931	4.971	0.662*	5.539+
	(0.581)	(0.095)	(1.930)	(4.699)	(0.295)	(2.898)
Num.Obs.	1879	1879	1879	1866	1866	1866
[0.5em] *PG*						
w.gdp:Other	-0.585	1.164+	-0.887	1.425	0.988	-0.082
	(0.631)	(0.593)	(0.807)	(1.180)	(0.920)	(0.510)
w.gdp:Post-92	8.567***	0.327	2.129	6.056**	0.688	1.803
	(1.733)	(0.349)	(2.028)	(2.025)	(0.461)	(1.418)
w.gdp:Pre-92, other	0.886	0.637*	-0.824	2.768	0.990*	1.563
	(1.243)	(0.322)	(0.877)	(2.270)	(0.472)	(1.041)
w.gdp:Pre-92, Russel group	-0.319	0.395	1.415	2.765	0.617	2.302+
	(5.351)	(0.321)	(4.571)	(3.486)	(0.489)	(1.329)
Num.Obs.	1944	1944	1944	1908	1908	1908
sample	All	All	All	Foreign	Foreign	Foreign

+ $p < 0.1$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

This figure and table shows the same as the previous, but with the coefficients split across university types. The results clearly distinguish between the Post-92 institutions relative to the others. Only Post-92 institutions expanded their offering in response to the Asian shock. The EM effect is larger to that of the other types of universities and statistically significant. The EM effect is also reflected in the outcome on the number of principal subjects (n.ps), while the effect in the intensive margin is much smaller.

5 Conclusion

- UG vs PG

Foreign students are more important for the PG level than for the UG level. We see this reflected in the results.

- Post-92 universities vs the rest

The former politecnical schools, called Post-92 institutions here, stood to be more substantially respond to the surge in foreign students. Relative to the older and established universities they were smaller and covered a smaller number of subjects. The results confirm that they increased their product scope most. The Russel group universities, instead, see much less of an effect. They may have increased student numbers, but their extensive coverage of subjects had little scope to increase further.

- Extensive margins

The surge of foreign students resulted mostly in an extensive margin effect.

Next steps

- Robustness and testing the identication strategy
- ...
- any suggestions welcome

A Model

To derive a gravity expression of a specific differentiated product, a two-country model with multi-product producing fimrs is presented. We follow Bernard et al (2010) and Hamano and Oikawa (2022) in modeling it. There is a mass J product categories in the economy. In product i, there is a mass $M_{i,t}$ of domestically produced product-varieties and $M^*_{Xi,t}$ of imported product varieties. Firms may or may not produce a specific product domestically. In the case of domestic provision, these prodcing firms may not export them due to a fixed costs for exporting.

We can interpret firms as "institutions" or "universities" which produce mulitple educaitonal services for domestic and foreign students.

A.1 Households

The representative household maximizes expected utility, $E_t \sum_{s=t}^{\infty} \beta^{s-t} U_t(j)$, where $0 < \beta < 1$ is the exogenous discount factor. The utility of each individual household j at time t depends on consumption $C_t(j)$ as follows:

$$U_t(j) = \frac{C_t(j)^{1-\gamma}}{1-\gamma},$$

where $\gamma \geq 1$ represents the risk aversion. Consumption consists of the number of exogenously given product categories J as

$$C_t(j) = \prod^{J} \left(\frac{C_{i,t}(j)}{\alpha_i} \right)^{\alpha_i},$$

where α_i stands for the preference weight on consumption of each product i.

The basket of each product i is defined over a continuum of "product varieties". During each period t, only a subset of product varieties $\Omega_{i,t}$ within Ω_i is available ($\Omega_{i,t} \subset \Omega_i$,). Product varieties are produced domestically or imported. They are indexed by $\omega \in \Omega_{i,t}$ for domestically produced product varieteis or $\omega^* \in \Omega_{i,t}$ for those imported. The basket of service product i is thus defined as the following CES aggregator:

$$C_{i,t}(j) = \left(\int_{\omega \in \Omega_t} \left(\lambda_i \left(j, \omega \right) c_{i,t} \left(j, \omega \right) \right)^{1-\frac{1}{\sigma_i}} d\omega + \int_{\omega^* \in \Omega_t} \left(\lambda_i^* \left(j, \omega^* \right) c_{Xi,t} \left(j, \omega^* \right) \right)^{1-\frac{1}{\sigma_i}} d\omega^* \right)^{\frac{1}{1-\frac{1}{\sigma_i}}},$$

where $c_{i,t}(\omega)$ and $c_{Xi,t}(\omega^*)$ stand for the demand for the domestically produced and imported product variety ω and ω^*, repectively. $\lambda_i(\omega)$ and $\lambda_i^*(\omega^*)$ represent the "taste" for these product varieties. $\sigma_i > 1$ stands for the elasticity of substitution among these product varieties. We assume that $\sigma_i > 0$. As a result of cost minimization, the optimal demand for domestically produced and imported product variety is found as

$$\lambda_i(\omega)\, c_{i,t}(\omega) = \left(\frac{p_{i,t}(\omega)\,/\,\lambda_i(\omega)}{P_{i,t}}\right)^{-\sigma_i} C_{i,t},\ \lambda_i^*(\omega^*)\, c_{Xi,t}(\omega^*) = \left(\frac{p_{Xi,t}^*(\omega^*)\,/\,\lambda_i^*(\omega^*)}{P_{i,t}}\right)^{-\sigma_i} C_{i,t} \tag{2}$$

where $p_{Di,t}(\omega)$ and $p_{Xi,t}^*(\omega^*)$ denote the price of variety indexed by ω and ω^*, respectively. Both prices are denominated in the home country unit.

Correspondingly, the price index of the product i is found as

$$P_{i,t} = \left(\int_{\omega\in\Omega_t}\left(\frac{p_{Di,t}(\omega)}{\lambda_i(\omega)}\right)^{1-\sigma_i} d\omega + \int_{\omega^*\in\Omega_t}\left(\frac{p_{Xi,t}(\omega^*)}{\lambda_i^*(\omega^*)}\right)^{1-\sigma_i} d\omega^*\right)^{\frac{1}{1-\sigma_i}}, \tag{3}$$

Furthermore, that the optimal demand for each product i is found to be

$$C_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-1} \alpha_i C_t \tag{4}$$

where price index of aggregate basket C_t is defined as

$$P_t = \prod^{J} P_{i,t}^{\alpha_i} \tag{5}$$

In what follows, we set P_t as numeraire.

Similar conditions hold for the foreign country.

A.2 Production, Pricing and Producing Decision

In the economy, there is a mass of N_t number of firms at time t. Firms are characterized by their idiocynclatic productivity levels, φ, which is drawn upon entry from a cumulative distribution, $G(\varphi)$ with support on $[\varphi_{\min}, \infty)$. Upon entry, firms also draw consumer taste level for each product, λ_i, from a cumulative distribution, $Z_i(\lambda_i)$ with support on $[\lambda_{i\min}, \infty)$.

Domestic production of product i by firm φ with taste λ_i, $y_{i,t}(\varphi, \lambda_i)$, requires fixed operational costs $f_{Di,t}$ as well as marginal costs both defined in labor units for every period. Furthermore, the amount $y_{Xi,t}(\varphi, \lambda_i)$ can be exported by incurring fixed costs $f_{Xi,t}$. We assume that $f_{Xi,t} > f_{Di,t}$. Total labor demand for an institution with productivity level φ is thus given by

$$l_t(\varphi) = \sum^{J}\left\{ I_i\left[\frac{y_{i,t}(\varphi,\lambda_i)}{\varphi} + f_{Di,t}\right] + I_{Xi}\left[\frac{y_{Xi,t}(\varphi,\lambda_i)}{\varphi} + f_{Xi,t}\right]\right\}. \tag{6}$$

I_i and I_{Xi} are indicators that take the unity if the firm produces and export the product i, otherwise 0.

Demand for product variety i is characterized by equation (2). Profit maximization by the firm φ with taste λ_i yields the following optimal real price:

$$\rho_{i,t}(\varphi, \lambda_i) = \frac{\sigma_i}{\sigma_i - 1}\frac{w_t}{\varphi}, \tag{7}$$

where $\rho_{i,t}(\varphi, \lambda_i)$ stands for the real price of service product i produced by the firm with productivity φ and consumer taste λ_i. w_t denotes real wages. Depending on the level of product-specific productivity, φ, and consumer taste λ_i, the product may or may not be produced. Thus, using equation (6), (7) and (4), if the firm provides the product, the following real operational firm-product specific profits are generated:

$$d_{Di,t}(\varphi, \lambda_i) = \frac{1}{\sigma_i}\left(\frac{\rho_{i,t}(\varphi,\lambda_i)}{\lambda_i \rho_{i,t}}\right)^{1-\sigma_i} \alpha_i C_t - w_{i,t} f_{Di,t}.$$

where $\rho_{i,t} \equiv \frac{P_{i,t}}{P_t}$, which is the real price of the basket of product i in the home country.

Because of the fixed operational costs, only products with $d_{Di,t}(\varphi, \lambda_i) > 0$ are domestically provided if consumer tastes λ_i are equal to or greater than $\lambda_{Di,t}(\varphi)$. For firm with productivity φ, there thus exists a zero profit consumer taste cutoff $\lambda_{Di,t}(\varphi)$ for product i. It is obtained by the following.

$$d_{Di,t}(\varphi, \lambda_{i,t}(\varphi)) = \frac{1}{\sigma_i}\left(\frac{\rho_{i,t}(\varphi, \lambda_i)}{\lambda_{i,t}(\varphi)\rho_{i,t}}\right)^{1-\sigma_i} \alpha_i C_t - w_{i,t} f_{Di,t} = 0. \tag{8}$$

Similary to the domesitc production, if the firm exports the service product, it generates the following operational profits from exporting as follows

$$d_{Xi,t}(\varphi, \lambda_i) = \frac{Q_t}{\sigma_i}\left(\frac{\rho_{Xi,t}(\varphi, \lambda_i)}{\lambda_i \rho^*_{i,t}}\right)^{1-\sigma_i} \alpha_i C^*_t - w_{i,t} f_{Xi,t}.$$

where $\rho_{Xi,t}(\varphi, \lambda_i) = Q_t^{-1}\tau\rho_{i,t}(\varphi, \lambda_i)$ is the real price of exporting of product i produced by the firm with productivity φ and consumer taste λ_i. Note that $\rho_{Xi,t}(\varphi, \lambda_i)$ is denominated in the foreign consumption units. In the expression of the real exporting price, Q_t stands for the real exchange rate defined as the price of foreign consumption in terms of home consumption units and τ represents the iceberg trade costs. $\rho^*_{i,t} \equiv \frac{P^*_{i,t}}{P^*_t}$ is the real price of the basket of product i in the foreign consumption units. Becuase of the fixed costs for exporting, the firm may not export this product variety. We thus define a cutoff taste from exporting $\lambda_{Xi,t}(\varphi)$ such that

$$d_{Xi,t}(\varphi, \lambda_{Xi,t}(\varphi)) = \frac{Q_t}{\sigma_i}\left(\frac{\rho_{Xi,t}(\varphi, \lambda_i)}{\lambda_{Xi,t}(\varphi)\rho^*_{i,t}}\right)^{1-\sigma_i} \alpha_i C^*_t - w_{i,t} f_{Xi,t} = 0.$$

Total operational profits of the institution with productivity φ is thus given by

$$d_t(\varphi) = \sum^{J} \left[I_i d_{Di,t}(\varphi, \lambda_i) + I_{Xi} d_{Xi,t}(\varphi, \lambda_i)\right].$$

A.3 Firm Entry and Exit

We assume that firmss entered at time t only start producing at time $t+1$. Firm entry occurs until the expected product value (9) is equal to the entry cost, leading to the following free entry condition:[1]

$$v_t = w_t f_{E,t} \tag{10}$$

where $f_{E,t}$ stands for the number of workers required upon entry. We denote the mass of entrants with H_t. The timing of entry and of production implies that the number of products evolves according to the law of motion:

$$N_t = (1-\delta)(N_{t-1} + H_{t-1}). \tag{11}$$

A.4 Product Averages

Two specific averages of productivities weighted by consumer tastes are defined following Bernard et al. (2010):

$$\widetilde{\varphi}_{i,t} \equiv \left[\int_{\varphi_{min}}^{\infty} \widetilde{\lambda}_{i,t}(\varphi) dG(\varphi)\right]^{\frac{1}{\sigma-1}}, \text{where} \quad \widetilde{\lambda}_{i,t}(\varphi) \equiv \int_{\lambda_{i,t}(\varphi)}^{\infty} (\lambda_i\varphi)^{\sigma-1} \frac{dZ_i(\lambda_i)}{1 - Z_i(\lambda_{i,t}(\varphi))}.$$

$$\widetilde{\varphi}_{Xi,t} \equiv \left[\int_{\varphi_{min}}^{\infty} \widetilde{\lambda}_{Xi,t}(\varphi) dG(\varphi)\right]^{\frac{1}{\sigma-1}}, \text{where} \quad \widetilde{\lambda}_{Xi,t}(\varphi) \equiv \int_{\lambda_{Xi,t}(\varphi)}^{\infty} (\lambda_i\varphi)^{\sigma-1} \frac{dZ_i(\lambda_i)}{1 - Z_i(\lambda_{Xi,t}(\varphi))}.$$

[1]The value of firms is discounted by the stream of their expected profits $\left\{\widetilde{d}_{s,k}\right\}_{k=t+1}^{\infty}$, using the stochastic discount factor of households adjusted by exogenous exit inducing shock δ. Thus, their expected post entry value is

$$v_t = E_t \sum_{k=t+1}^{\infty} [\beta(1-\delta)]^{k-t}\left(\frac{\Lambda_t}{\Lambda_k}\right)\widetilde{d}_k, \tag{9}$$

which represents the share price of equities and mutual funds across different products. Λ_k is a discount factor of the representative household which is defined below.

In the above expression $\widetilde{\lambda}_{i,t}(\varphi)$ and $\widetilde{\lambda}_{Xi,t}(\varphi)$ stand for the average taste of product i for the firm with productivity φ in domestic and exporting markets. They summarize the space of the taste that is capable for production of product i by the firm with φ. The term, $\widetilde{\varphi}_{i,t}$ thus contains all the information about the distribution of productivities and also consumer tastes. In short, these specific averages can be interpreted as the taste-weighted-average productivities of service product i in domestic and exporting markets. Using the taste weighted average productivity in domestic market, the taste-adjusted real price for product i of the average producer is defined as

$$\widetilde{\rho}_{i,t} = \frac{\sigma_i}{\sigma_i - 1} \frac{w_{i,t}}{\widetilde{\varphi}_{i,t}}.$$

Based on this real price, we can now define the average profits in domestic market for each product i as

$$\widetilde{d}_{Di,t} = \frac{1}{\sigma_i} \left(\frac{\tilde{\rho}_{Di,t}}{\rho_{i,t}} \right)^{1-\sigma_i} \alpha_i C_t - w_{i,t} f_{Di,t} \tag{12}$$

And those from exporting are defined as

$$\tilde{d}_{Xi,t} = \frac{Q_t}{\sigma_i} \left(\frac{\tilde{\rho}_{Xi,t}}{\rho^*_{i,t}} \right)^{1-\sigma_i} \alpha_i C^*_t - w_{i,t} f_{Xi,t}$$

Further, total profits from the product i is found to be

$$\widetilde{d}_{i,t} = \widetilde{d}_{Di,t} + \frac{M_{Xi,t}}{M_{i,t}} \tilde{d}_{Xi,t}$$

Finally, the average real profits of all producer firms are expressed as follows:

$$\widetilde{d}_t = \sum^{J} \frac{M_{i,t}}{N_t} \widetilde{d}_{i,t} \tag{13}$$

where$M_{i,t} = \int_{\varphi_{min}}^{\infty} \left[1 - Z_i \left(\lambda_{i,t}\left(\varphi\right)\right)\right] dG(\varphi) N_t$ and $M_{Xi,t} = \int_{\varphi_{min}}^{\infty} \left[1 - Z_i \left(\lambda_{Xi,t}\left(\varphi\right)\right)\right] dG(\varphi) N_t$ denote the number of domestically serving firms and the number of firms that export their product varieties.

Similar expressions hold in the foreign country.

A.5 Parametrization of Productivity and Taste Draw

To solve the model, we assume a distribution of productivity levels, φ and λ_i. Specifically, the following Pareto distributions for $G(\varphi)$ and $Z_i(\lambda_i)$ are considerd:

$$G(\varphi) = 1 - \left(\frac{\varphi_{\min}}{\varphi} \right)^{\kappa}, \quad Z_i(\lambda_i) = 1 - \left(\frac{\lambda_{i\,\min}}{\lambda_i} \right)^{v}$$

where $\varphi_{\min}$ and $\lambda_{i\,\min}$ are the minimum productivity level. κ and v are the parameters that determine the shape of these distributions. The dispersion decreases as these parameters increase, and the productivity or tastes are concentrated toward the lower bound $\varphi_{\min}$ and $\lambda_{i\,\min}$. We set $\varphi_{\min} = \lambda_{i\,\min} = 1$ without loss of generality. To ensure that variance of the productivity distribution are finite and that the number of products is positive, we assume that $\kappa > v > \sigma - 1$. With this parametrization, we can express the taste-weighted-average productivity $\widetilde{\varphi}_{i,t}$ as[2]

$$\widetilde{\varphi}_{i,t} = \left[\frac{v}{v - (\sigma_i - 1)} \right]^{\frac{1}{\sigma_i - 1}} \varphi_{min} \lambda_{Di,t}(\varphi_{min}), \tag{14}$$

[2]Using the zero profits consumer taste cutoff (8) for the institution with productivity $\varphi_{\min}$, the consumer taste cutoff of establishment with productivity φ_t, i.e., $\lambda_{i,t}(\varphi)$, can be expressed as a function of cutoff productivity level $\varphi_{\min}$ and the consumer taste cutoff of this minimum productivity institution $\lambda_{i,t}(\varphi_{\min})$ as $\lambda_{i,t}(\varphi) = \frac{\varphi_{\min}}{\varphi} \lambda_{i,t}(\varphi_{\min})$. The expression has an intuitive interpretation. The cutoff consumer taste of the institution is decreasing with respect to her own productivity because it allows to produce even with a lower end of taste. It is increasing with respect to $\varphi_{\min}$ and $\lambda_{i,t}(\varphi_{\min})$ since a higher value of each intensifies the competition. The above characteristic in turn means that the average taste-weighted productivity $\widetilde{\varphi}_{i,t}$ is expressed in terms of $\varphi_{\min}$ and $\lambda_{i,t}(\varphi_{\min})$. Specifically, with the Pareto distribution as in the paper, $\widetilde{\lambda}_{Di,t}(\varphi) = \frac{v}{v-(\sigma_i-1)} \left[\varphi_{\min}\lambda_{i,t}(\varphi_{\min})\right]^{\sigma_i - 1}$ and thus $\widetilde{\varphi}_{i,t} = \int_{\varphi_{\min}}^{\infty} \widetilde{\lambda}_{i,t}(\varphi) dG(\varphi) = \frac{v}{v-(\sigma_i-1)} \left[\varphi_{\min}\lambda_{i,t}(\varphi_{\min})\right]^{\sigma_i-1} \int_{\varphi_{\min}}^{\infty} dG(\varphi) = \frac{v}{v-(\sigma_i-1)} \left[\varphi_{\min}\lambda_{i,t}(\varphi_{\min})\right]^{\sigma_i-1}$. A similar argument holds for $\widetilde{\varphi}_{Xi,t}^{\sigma_i-1}$.

and the fraction of surviving products as

$$\frac{M_{i,t}}{N_t} = \frac{\kappa}{\kappa - v} \lambda_{Di,t} \left(\varphi_{min} \right)^{-v}, . \tag{15}$$

By combining (14) and (15), we get

$$\widetilde{\varphi}_{i,t} = \left[\frac{v}{v - (\sigma_i - 1)} \right]^{\frac{1}{\sigma_i - 1}} \left(\frac{M_{i,t}}{N_t} \frac{\kappa - v}{\kappa} \right)^{-\frac{1}{v}}. \tag{16}$$

As mentioned earlier, for the firm with the cutoff level productivity, we can define the zero profit consumer taste cutoff condition as $d_{Di,t} \left(\varphi, \lambda_{i,t} \left(\varphi \right) \right) = 0$. This implies that

$$\widetilde{d}_{Di,t} = \frac{\sigma_i - 1}{v - (\sigma_i - 1)} w_{i,t} f_{Di,t}. \tag{17}$$

Also for exporting institutions, we have a similar taste-weighted average productivity as

$$\widetilde{\varphi}_{Xi,t} = \left[\frac{v}{v - (\sigma_i - 1)} \right]^{\frac{1}{\sigma - 1}} \left(\frac{M_{Xi,t}}{N_t} \frac{\kappa - v}{\kappa} \right)^{-\frac{1}{v}}. \tag{18}$$

and from $d_{Xi,t} \left(\varphi, \lambda_{Xi,t} \left(\varphi \right) \right) = 0$, we have

$$\widetilde{d}_{Xi,t} = \frac{\sigma_i - 1}{v - (\sigma_i - 1)} w_{i,t} f_{Xi,t}. \tag{19}$$

Similar conditions hold in the foreign country.

A.6 Household Budget Constraints and Intertemporal Problems

The household receives income by exogenously supplying labor $L(j)$, at the real wage rate w_t, by acquiring average dividends income $\widetilde{d}_t$, and by selling its initial share position v_t, of shareholdings x_t, of the firm composed of existing products, N_t. The household spends its income on consumption $C_t(j)$, buying $x_{t+1}(j)$ shares of the firm composed of existing products N_t, and new products H_t, at share price v_t. The household budget constraint is thus

$$L_t(j) w_t + x_t(j) N_t \left(v_t + \widetilde{d}_t \right) + Q_t = C_t(j) + x_{t+1}(j) v_t \left(N_t + H_t \right). \tag{20}$$

During each period t, the representative household chooses consumption $C_t(j)$, shareholdings $x_{t+1}(j)$, and the labor supply $L_t(j)$, to maximize the expected utility function subject to the budget constraint (20).

The first-order condition with respect to shareholdings once combined with the product law of motion (11) and the first-order condition for consumption yields

$$v_t = \beta \left(1 - \delta \right) E_t \left[\frac{\Lambda_{t+1}(j)}{\Lambda_t(j)} \left(v_{t+1} + \widetilde{d}_{t+1} \right) \right], \tag{21}$$

where $\Lambda_t(j) = C_t(j)^{-\gamma}$ stands for the shadow value of the budget constraint. which, once iterated forward, shows that share prices are the expected discounted sum of future dividends.

Similar conditions hold for the foreign country.

A.7 Model equilibrium and solution

In equilibrium, the households symmetrically chose the same level of consumptions. As a resutls, we have $C_t(j) = C_t$, $C_{i,t}(j) = C_{i,t}$, and $L(j) = L$. Also, in equiliburium, labor market clears. Aggregate labor supply for each service product, L_i, is employed in either the production or fixed operational costs in domesic and export markets as follows:

$$L = \sum^{J} L_i$$

where

$$L_i = M_{i,t}\left(\frac{\widetilde{y}_{Di,t}}{\widetilde{\varphi}_{i,t}} + f_{Di,t}\right) + M_{Xi,t}\left(\frac{\widetilde{y}_{Xi,t}}{\widetilde{\varphi}_{Xi,t}} + f_{Xi,t}\right),$$

The condition can be expressed as[3]

$$L = \sum^{J} M_{i,t}\left[(\sigma_i - 1)\,\frac{\widetilde{d}_{Di,t}}{w_t} + \sigma_i f_{Di,t}\right] + M_{Xi,t}\left[(\sigma_i - 1)\,\frac{\widetilde{d}_{Xi,t}}{w_t} + \sigma_i f_{Xi,t}\right]. \tag{22}$$

Equation (22) is equivalent to the aggregated accounting identity of GDP obtained by aggregating budget constraints among households.

Similar conditions hold in the foreign country.

Finally, we assume that international financial markets is complete due to the internationally traded state-contignet claims (while we do not explicitly model these securities). As a result, the marginal utility from one unit of nominal wealth becomes the same across country leading to the following condition under the complete financial markets:

$$C_t^{-\gamma} Q_t = C_t^{*-\gamma}$$

For a particuler product i, there are 33 equations and 33 endogenous variables which are $\widetilde{\rho}_{i,t}$, $\widetilde{\varphi}_{i,t}$, w_t, $\widetilde{\rho}_{Xi,t}$, $\widetilde{\varphi}_{Xi,t}$, $\rho_{i,t}$, $M_{i,t}$, $M_{Xi,t}$, $\widetilde{d}_{Di,t}$, $\widetilde{d}_{Xi,t}$, $\widetilde{d}_{i,t}$, $\widetilde{d}_t$, C_t, v_t, H_t, N_t, $\widetilde{\rho}^*_{Di,t}$, $\widetilde{\varphi}_{i,t}$, w^*_t, $\widetilde{\rho}^*_{Xi,t}$, $\widetilde{\varphi}^*_{Xi,t}$, $\rho^*_{i,t}$, $M_{i,t}$, $M^*_{Xi,t}$, $\widetilde{d}^*_{Di,t}$, $\widetilde{d}^*_{Xi,t}$, $\widetilde{d}^*_{i,t}$, $\widetilde{d}^*_t$, C^*_t, v^*_t, H^*_t, N^*_t, and Q_t. Table 1 summarizes the benchmark simple model. To save the space, we show only for the system for the home country.

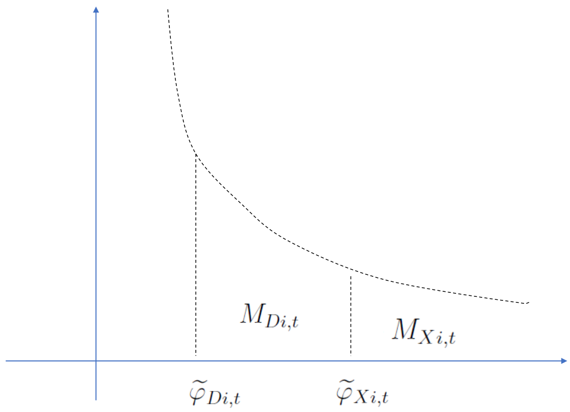
Table 2: Summary of the model (given foreign variables)

1. Average real price ($\times J$)	$\widetilde{\rho}_{i,t} = \frac{\sigma_i}{\sigma_i - 1}\frac{w_t}{\widetilde{\varphi}_{i,t}}$
2. Average real price for exporting ($\times J$)	$\widetilde{\rho}_{Xi,t} = \tau Q_t^{-1}\frac{\sigma_i}{\sigma_i - 1}\frac{w_t}{\widetilde{\varphi}_{Xi,t}}$
3. Real taste-adjusted price ($\times J$)	$\rho_{i,t}^{1-\sigma_i} = M_{i,t}\tilde{\rho}_{i,t}^{1-\sigma_i} + M^*_{Xi,t}\tilde{\rho}^{*1-\sigma_i}_{Xi,t}$
4. Price index	$1 = \sum^J \rho_i^{\alpha_i}$
5. Average profits from domestic sales ($\times J$)	$\widetilde{d}_{Di,t} = \frac{1}{\sigma_i}\left(\frac{\tilde{\rho}_{i,t}}{\rho_{i,t}}\right)^{1-\sigma_i}\alpha_i C_t - w_t f_{Di,t}$
6. Average product profits from exporting ($\times J$)	$_{Xi,t} = \frac{Q_t}{\sigma_i}\left(\frac{\tilde{\rho}_{Xi,t}}{\rho^*_{i,t}}\right)^{1-\sigma_i}\alpha_i C^*_t - w_t f_{Xi,t}$
7. Average product profits ($\times J$)	$\widetilde{d}_{i,t} = \widetilde{d}_{Di,t} + \frac{M_{Xi,t}}{M_{i,t}}\widetilde{d}_{Xi,t}$
7. Average profits of firms	$\widetilde{d}_t = \sum^J \widetilde{d}_{i,t}$
9. Cutoff for domestic production($\times J$)	$\widetilde{d}_{Di,t} = \frac{\sigma_i - 1}{v - (\sigma_i - 1)} w_t f_{Di,t}$
10. Cutoff for exporting ($\times J$)	$\widetilde{d}_{Xi,t} = \frac{\sigma_i - 1}{v - (\sigma_i - 1)} w_t f_{Xi,t}$
11. Average taste weighted productivity ($\times J$)	$\widetilde{\varphi}_{i,t} = \left[\frac{v}{v - (\sigma_i - 1)}\right]^{\frac{1}{\sigma_i - 1}}\left(\frac{M_{i,t}}{N_t}\frac{\kappa - v}{\kappa}\right)^{-\frac{1}{v}}$
12. Average taste weighted productivity for exporters ($\times J$)	$\widetilde{\varphi}_{Xi,t} = \left[\frac{v}{v - (\sigma_i - 1)}\right]^{\frac{1}{\sigma_i - 1}}\left(\frac{M_{Xi,t}}{N_t}\frac{\kappa - v}{\kappa}\right)^{-\frac{1}{v}}$
13. Labor market clearing	$L = \sum^J \left\{ M_{i,t}\left[(\sigma_i - 1)\frac{\widetilde{d}_{Di,t}}{w_t} + \sigma_i f_{Di,t}\right] + \right.$
	$\left. M_{Xi,t}\left[(\sigma_i - 1)\frac{\widetilde{d}_{Xi,t}}{w_t} + \sigma_i f_{Xi,t}\right]\right\}$
14. Euler equation	$v_t = \beta(1-\delta)E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\left(v_{t+1} + \widetilde{d}_{t+1}\right)\right]$
15. Free entry condition	$v_t = w_t f_{E,t}$
16. Motion of firms	$N_{t+1} = (1-\delta)(N_t + H_t)$
17. Complete market condition	$C_t^{-\gamma} Q_t = C_t^{*-\gamma}$

Figure 1 below shows the configuration in a particuler product i. Note that by assuming that $f_{Xi,t} > f_{Di,t}$, it is plausible to have $\widetilde{\varphi}_{i,t} < \widetilde{\varphi}_{Xi,t}$. Thus, some product varieties are only offered domestically.

[3] Note that $\widetilde{d}_{Di,t} = \frac{\widetilde{\rho}_{i,t}}{\sigma}\widetilde{y}_{Di.t} - \frac{w_t f_{i,t}}{Z_t}$.

Figure 1: Configuration of subject area i



B Gravity Expressions

B.1 Intensive and Extensive margins

Increasing foreign demand captured by rising C_t^* directly impacts the profits from exporting as

$$\tilde{d}_{Xi,t} = \frac{Q_t}{\sigma_i}\left(\frac{\tilde{\rho}_{Xi,t}}{\rho^*_{i,t}}\right)^{1-\sigma_i} \alpha_i C_t^* - w_t f_{Xi,t}$$

By plugging the equations in Table 1 we can rewrite the above expression as

$$\frac{M_{Xi,t}}{N_t} = \Theta\left(\frac{\rho^*_{i,t}}{\tau}\right)^{v}\left(\frac{Q_t}{w_t}\right)^{v\frac{\sigma_i}{\sigma_i-1}}\left(\frac{\alpha_i C_t^*}{f_{Xi,t}}\right)^{\frac{v}{\sigma_i-1}} \tag{23}$$

where $\Theta \equiv \frac{\kappa}{\kappa-v}\left(\frac{\sigma_i-1}{\sigma_i}\right)^{v}/\sigma_i^{\frac{v}{\sigma_i-1}}$. Furthermore, we have the following expression for the average real sales of service product variety i (in domestic consumption units) as[4]

$$Q_t\widetilde{\rho}_{Xi,t}\widetilde{y}_{Xi.t} = \sigma_i\left(\frac{v}{v-(\sigma_i-1)}\right)w_t f_{Xi,t} \tag{24}$$

Combining (23) and (24), we get the average real value of exports per firm (in foreign consumption units) as

$$\frac{M_{Xi,t}\widetilde{\rho}_{Xi,t}\widetilde{y}_{Xi.t}}{N_t} = \Delta\left(\frac{\rho^*_{i,t}}{\tau}\right)^{v}\left(\frac{Q_t}{w_t}\right)^{v\frac{\sigma_i}{\sigma_i-1}-1}(\alpha_i C_t^*)^{\frac{v}{\sigma_i-1}} f_{Xi,t}^{-\left(\frac{v}{\sigma_i-1}-1\right)} \tag{25}$$

where $\Delta \equiv \frac{(\sigma_i-1)^{v}}{\sigma_i^{v\frac{\sigma_i}{\sigma_i-1}-1}}\frac{v}{v-(\sigma_i-1)}\frac{\kappa}{\kappa-v}$. The expression (25) is a gravity equation with respect to a particuler product i. It shares the same features discussed in Chaney (2009) and Hamano and Vermeulen (2022). The real export value per firms (in foreign consumption units) is increasing with respect to foreign aggregate demand C_t^*, preference for the subject area α_i, real exchange rate Q_t, and the extent of competition in destination captured by price index $\rho^*_{i,t}$. It decreases with respect to higher domestic costs represented by w_t, trade costs τ, and fixed costs for exporting $f_{Xi,t}$.

[4]Note that we have

$$\widetilde{y}_{Xi.t} = (\sigma_i-1)\left(\frac{v}{v-(\sigma_i-1)}\right)^{1+\frac{1}{\sigma_i-1}}\left(\frac{\kappa-v}{\kappa}\right)^{-\frac{1}{v}}\left(\frac{M_{Xi,t}}{N}\right)^{-\frac{1}{v}}\frac{f_{Xi,t}}{\tau}$$

So the scale of production $\widetilde{y}_{Xi.t}$ and the extensive margins of trade $\frac{M_{Xi,t}}{N}$ are negatively related.

Also we get similar expressions for domestic extensive margins and intensive margins as

$$\frac{M_{i,t}}{N_t} = \Theta \rho_{i,t}^{v} \left(\frac{1}{w_t}\right)^{v\frac{\sigma_i}{\sigma_i - 1}} \left(\frac{\alpha_i C_t}{f_{Di,t}}\right)^{\frac{v}{\sigma_i - 1}} \tag{26}$$

and

$$\widetilde{\rho}_{Di,t}\widetilde{y}_{Di.t} = \frac{v\sigma_i}{v - (\sigma_i - 1)} w_t f_{Di,t} \tag{27}$$

Thus, by combining (26) and (27), the real value of domestic average sales for product variety i is

$$\frac{M_{i,t}\widetilde{\rho}_{Di,t}\widetilde{y}_{Di.t}}{N_t} = \Delta \rho_{i,t}^{v} w_t^{-\left(v\frac{\sigma_i}{\sigma_i - 1} - 1\right)} \left(\alpha_i C_t\right)^{\frac{v}{\sigma_i - 1}} f_{Di,t}^{-\left(\frac{v}{\sigma_i - 1} - 1\right)} \tag{28}$$

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